



**WELCOME**

14-June -2023



## **Session 8**

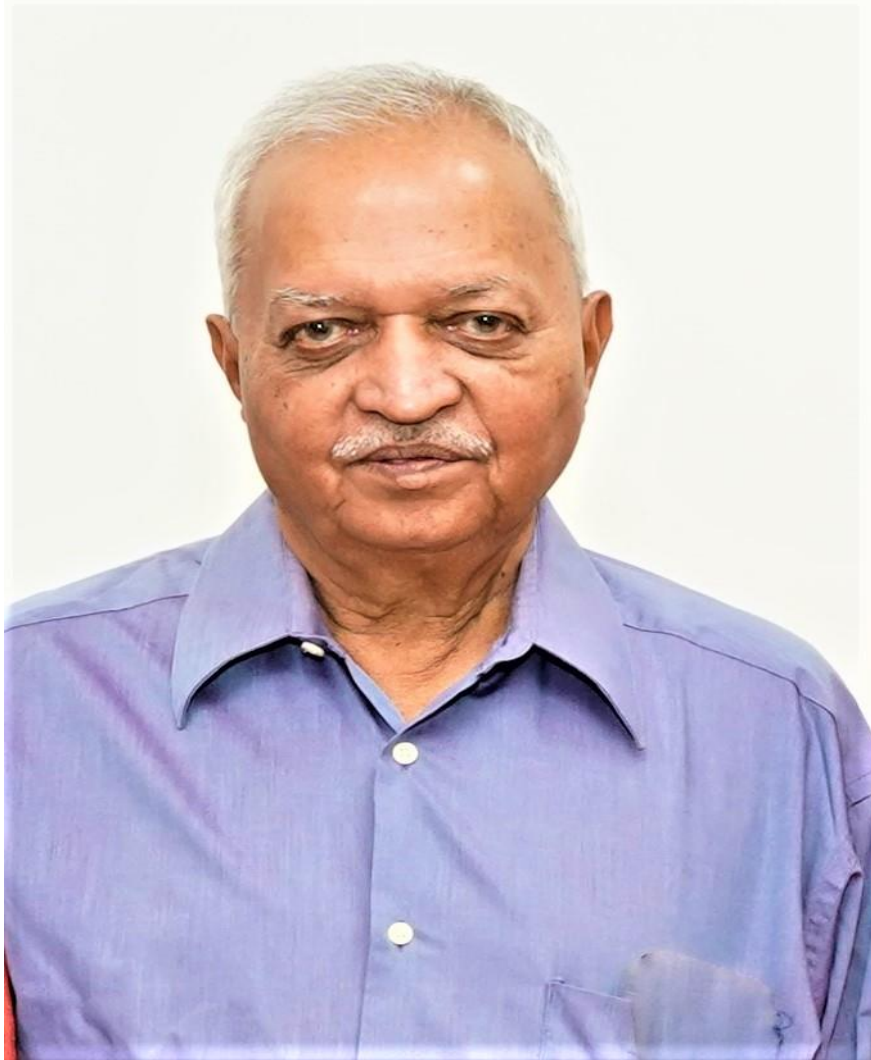
# **Introduction and Basics of Seismic Analysis of Structures, including Response Spectrum Method**

**By**

**Prof. M. G. Gadgil**

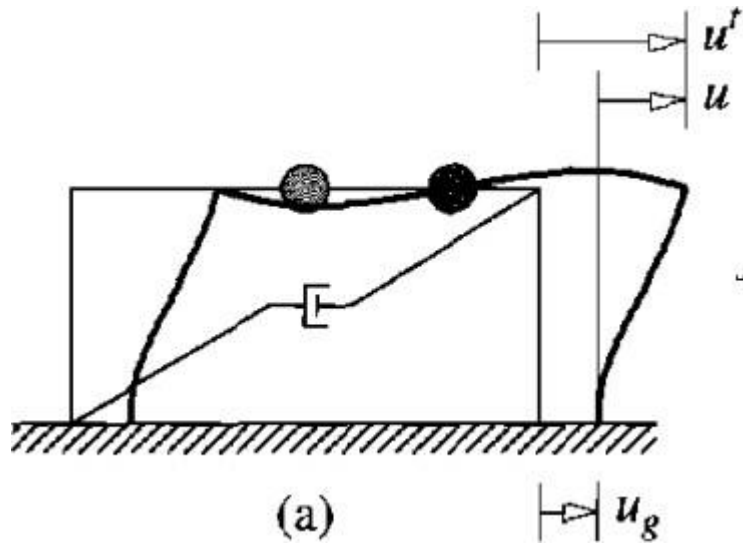


# Prof. Manohar G Gadgil



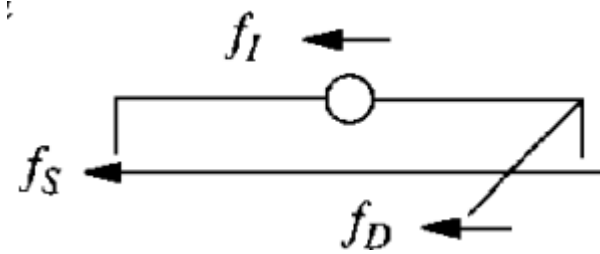
- Prof. Manohar Gadgil is retired professor from VJTI.
- He was HOD of the structural department of VJTI.
- He has completed his Bachelor of Engineering in Civil from the University of Bombay in 1970 and M. Tech. in Structure from I.I.T. Powai in 1975.
- He has published several papers at Indian and international conferences.
- During the last 33 years, he has guided more than 100 P.G. students in their dissertation work.
- The software needed for the projects was developed by him in the days when ready-to-use software was not available on the market.
- He is providing consultancy services for high-rise buildings & various industrial building structures.
- ❖ Design of steel structures such as canopies, domes, pyramids, skylights, glazing
- ❖ Design of glazing works on various office/commercial buildings
- ❖ Design of pre-stressed concrete floor grid systems & floor slabs
- ❖ Design of special hinge and roller supports
- ❖ Design of pre-engineered offices
- ❖ Design of precast buildings
- ❖ Development of several testing facilities for testing building materials etc.





$$u^t(t) = u(t) + u_g(t)$$

Total disp = relative disp + ground disp



Equation of motion is given by

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

Dividing above equation by m we get the following equation

$$\ddot{u} + 2\zeta\omega_n \dot{u} + \omega_n^2 u = -\ddot{u}_g(t)$$

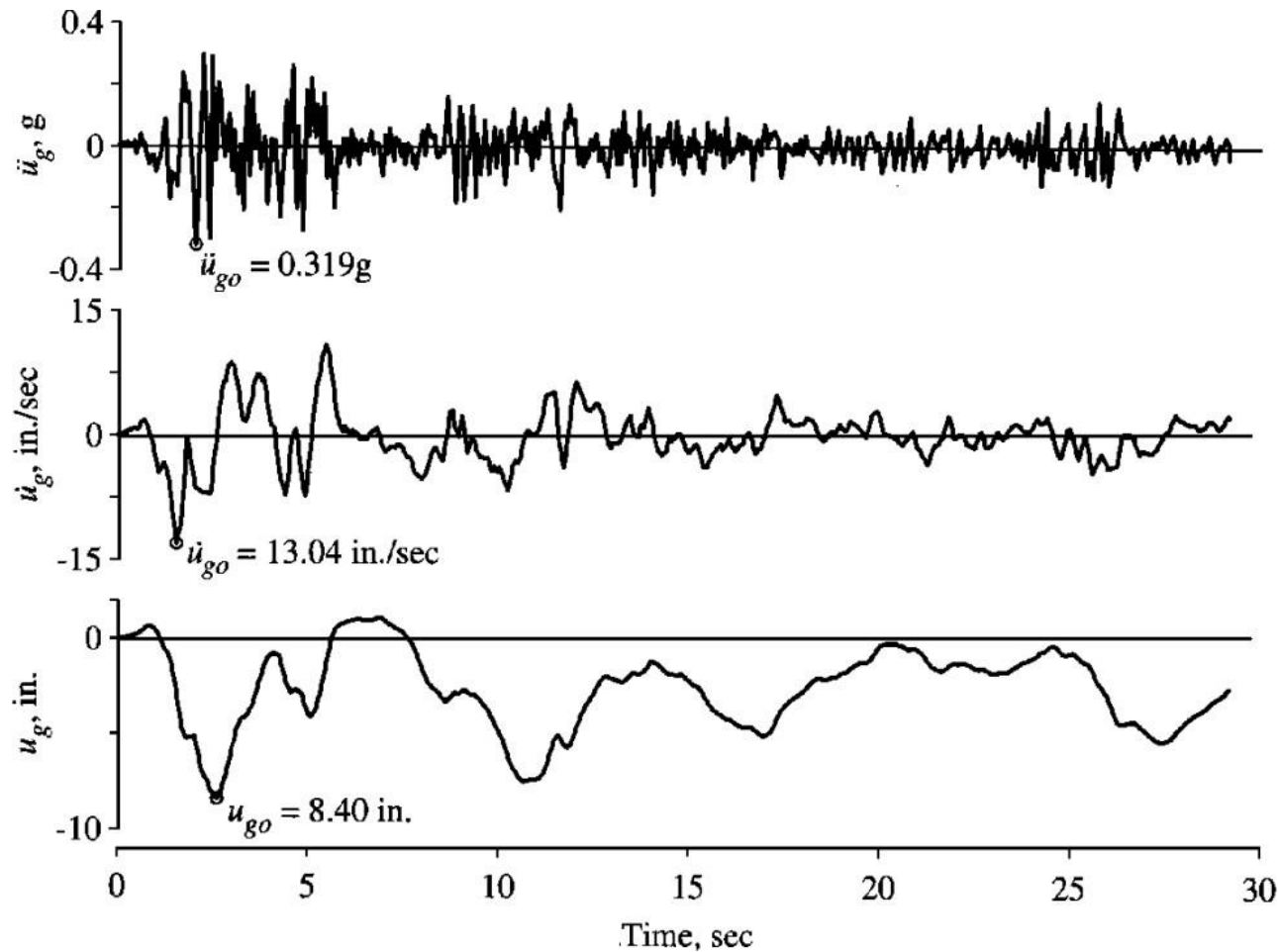
It is clear that for a given  $\ddot{u}_g(t)$ , the deformation response  $u(t)$  of the system depends only on the natural frequency  $\omega_n$  or natural period  $T_n$  of the system and its damping ratio  $\zeta$ ;

In other words  $u \equiv u(t, T_n, \zeta)$ .

**Thus any two systems having the same values of**

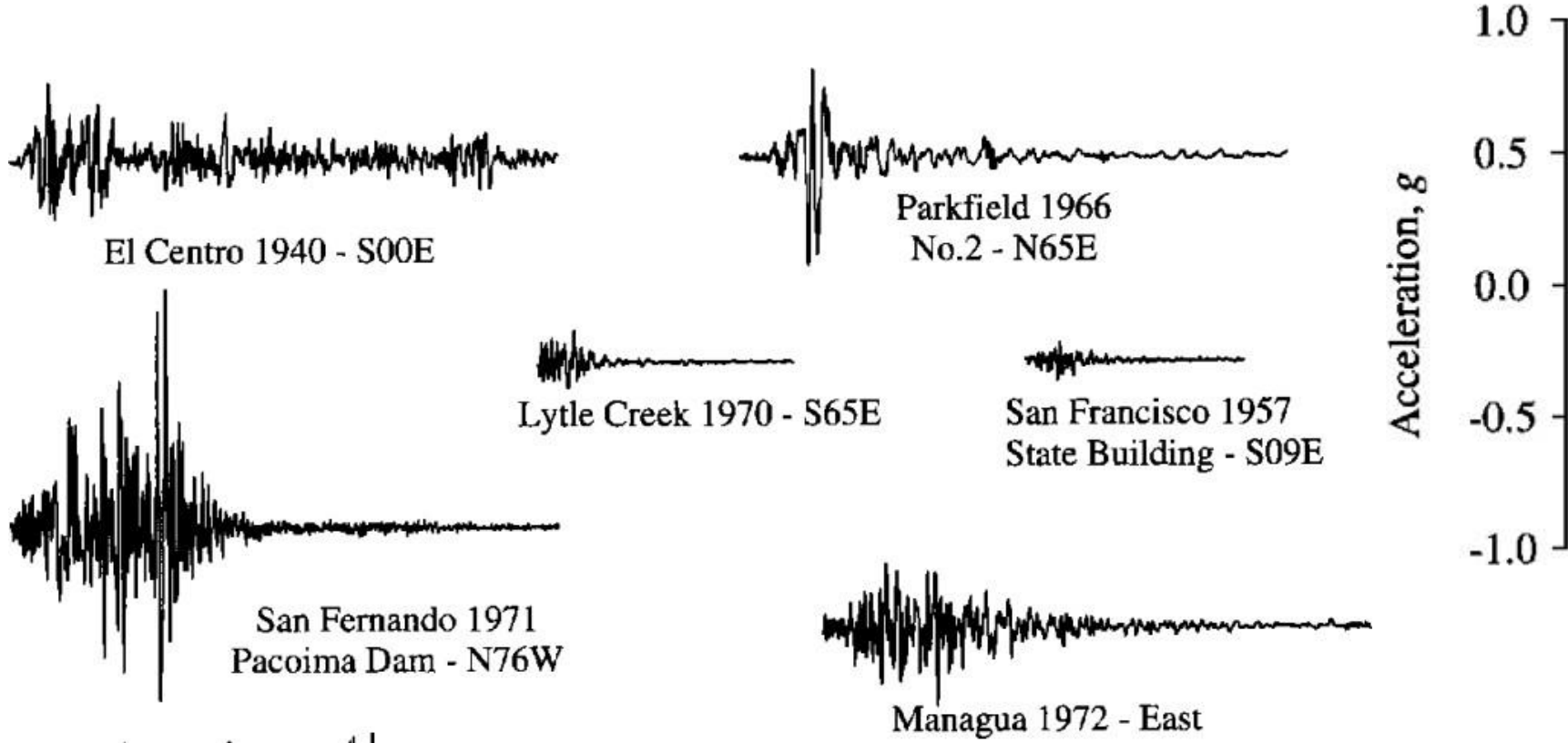
$T_n$  and  $\zeta$  will have the same deformation response  $u(t)$  even though one system may be more massive than the other or one may be stiffer than the other.

# Typical Ground Motion Records



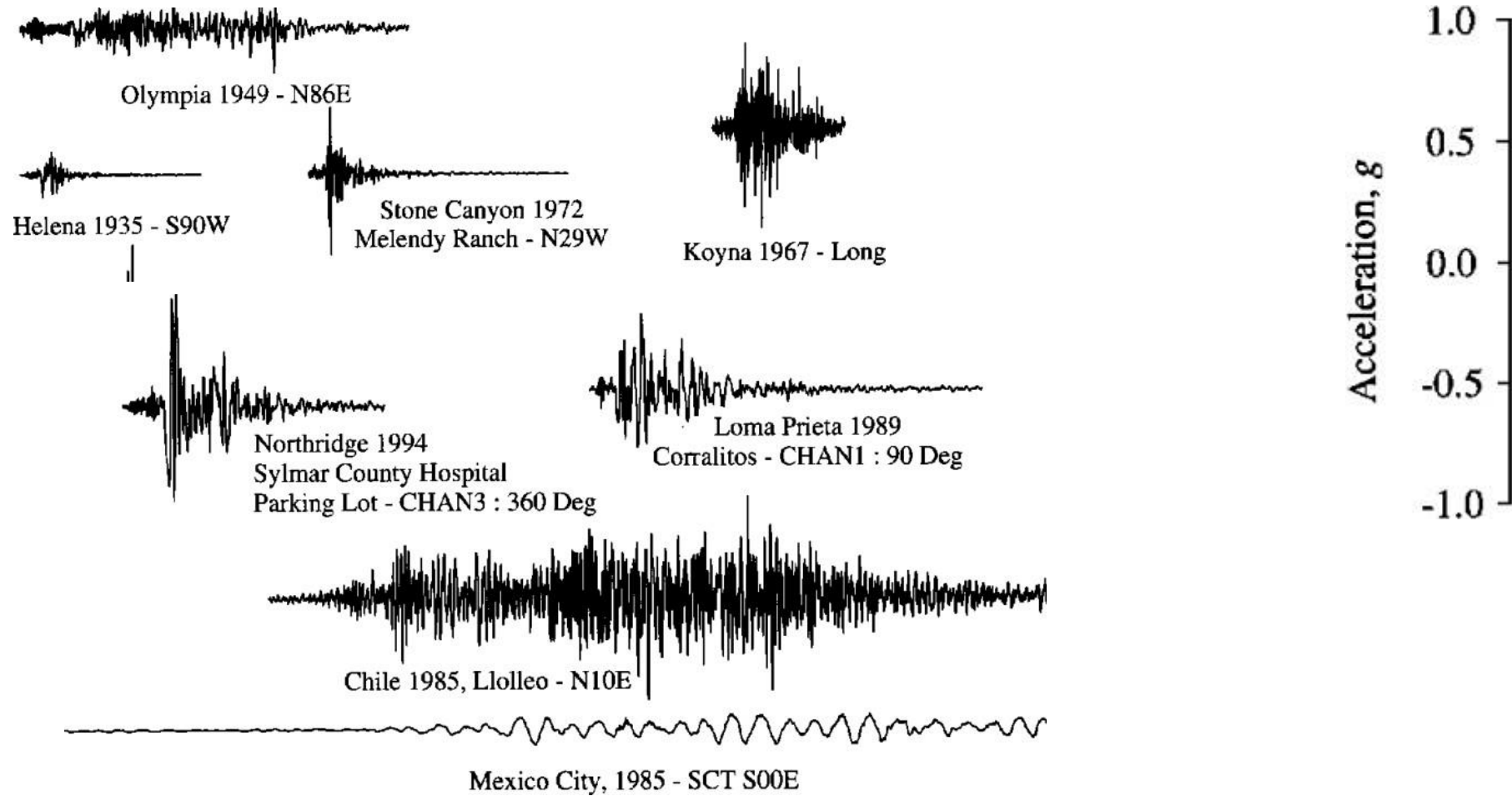
North-south component of horizontal ground acceleration recorded at the Imperial Valley Irrigation District substation, El Centro, California, during the Imperial Valley earthquake of May 18, 1940. The ground velocity and ground displacement were computed by integrating the ground acceleration.

# Earthquake response of linear system



Ground motion recorded during several earthquakes

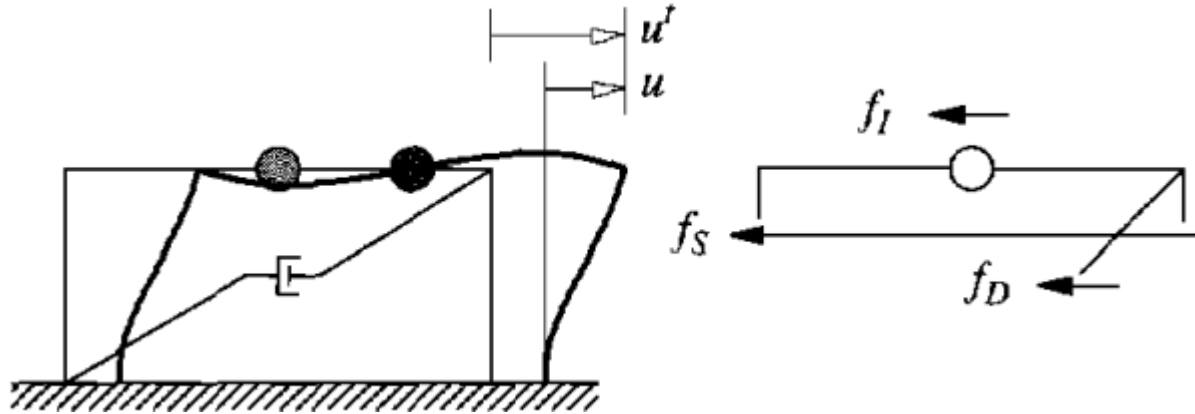
# Earthquake response of linear system contd.



Ground motion recorded during several earthquakes



# Simple system under ground motion

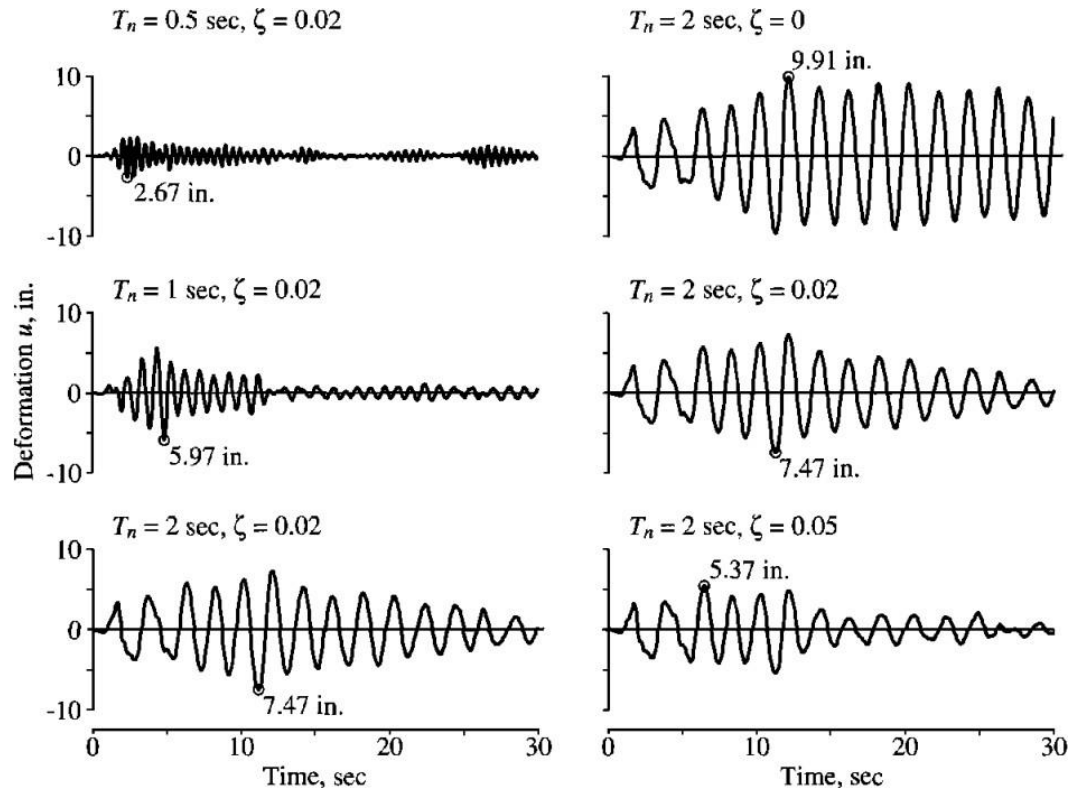


Equation of motion under the three forces is

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

# Response of SDOF to ground motion

For a given ground motion  $\ddot{u}_g(t)$ , the deformation response  $u(t)$  of an SDF system depends only on the natural vibration period of the system and its damping ratio.



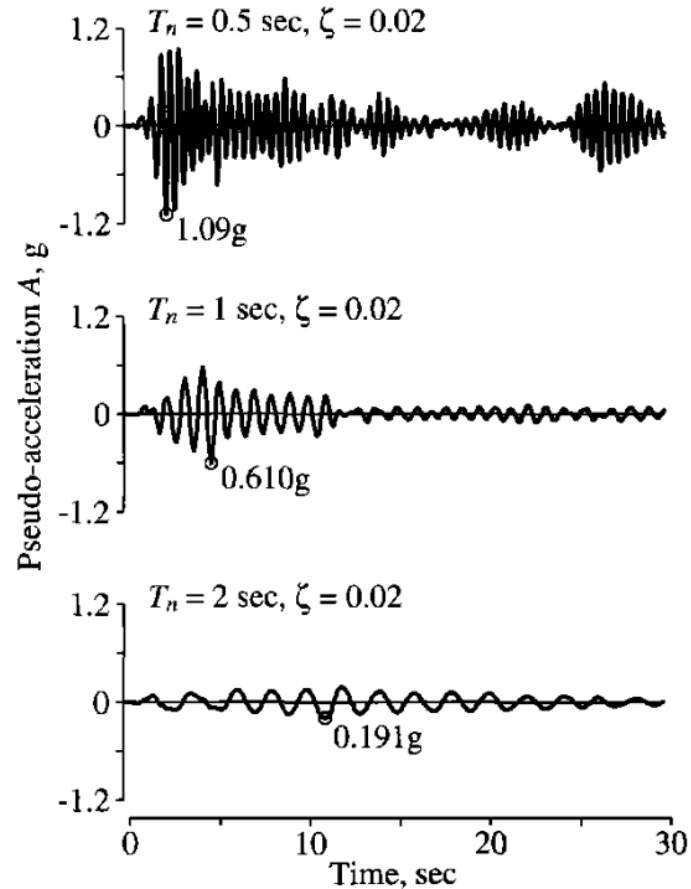
Deformation response of SDF systems to El Centro ground motion.

# Pseudo acceleration response

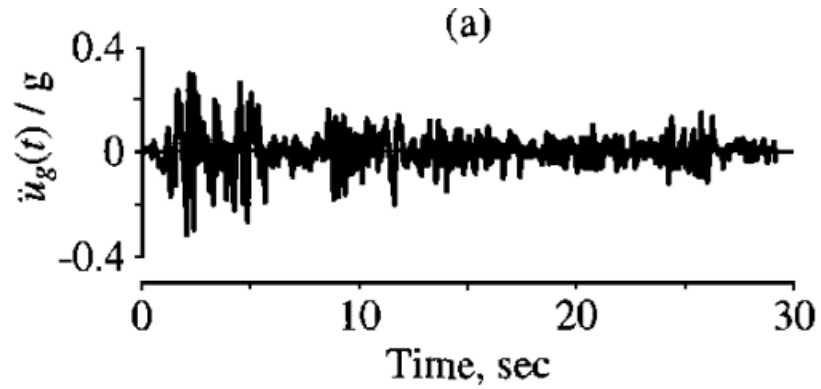
Pseudo acceleration = displacement  $\times \omega_n^2 = \left(\frac{2\pi}{T_n}\right)^2$

$$V = \omega_n D = \frac{2\pi}{T_n} D$$

$$A = \omega_n^2 D = \left(\frac{2\pi}{T_n}\right)^2 D$$

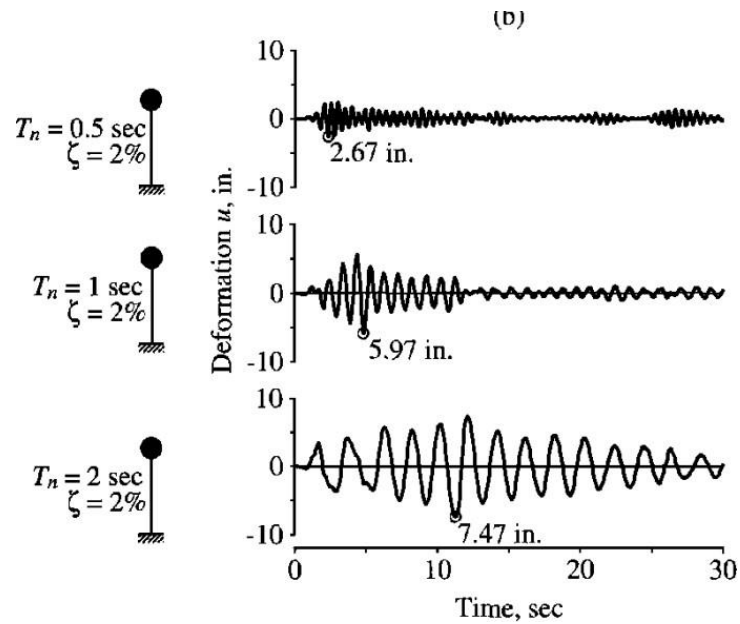


# Response spectrum

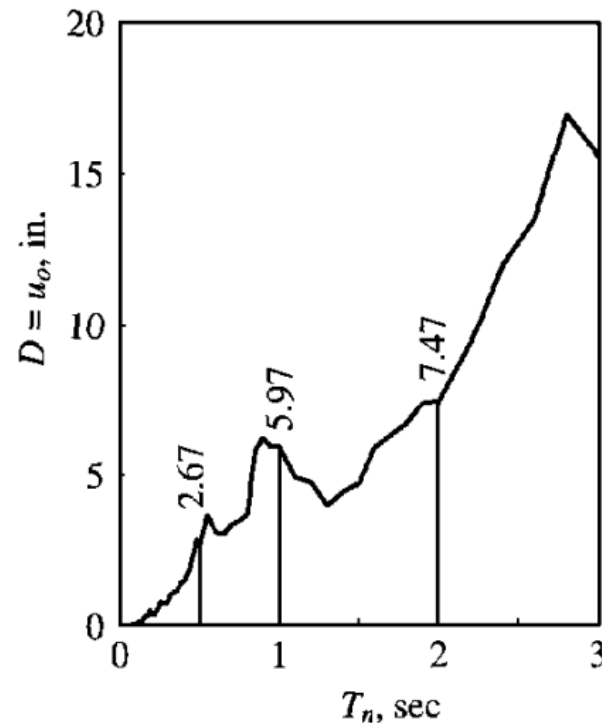


El-centro ground motion

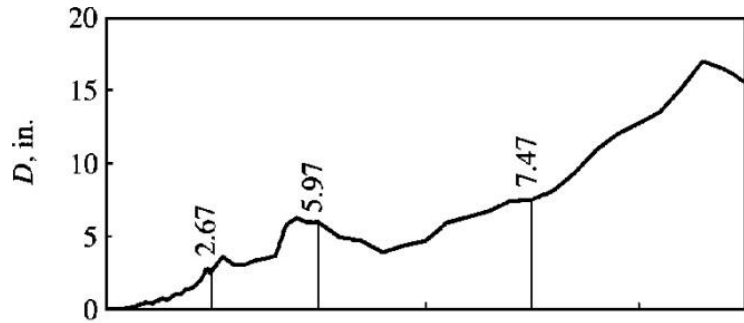
Deformation response spectrum



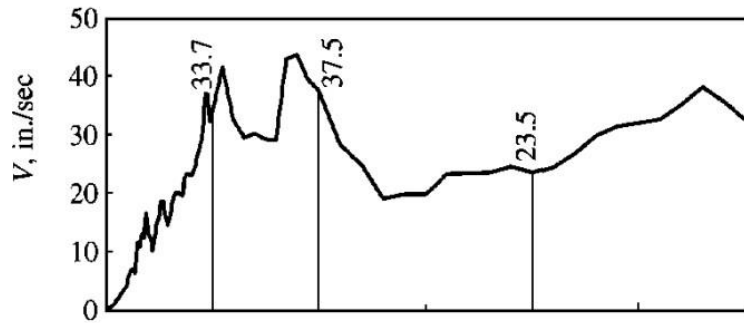
Deformation response



# Pseudo velocity and pseudo acceleration spectra



Deformation Response Spectrum  
Gives force  $F = k \Delta$

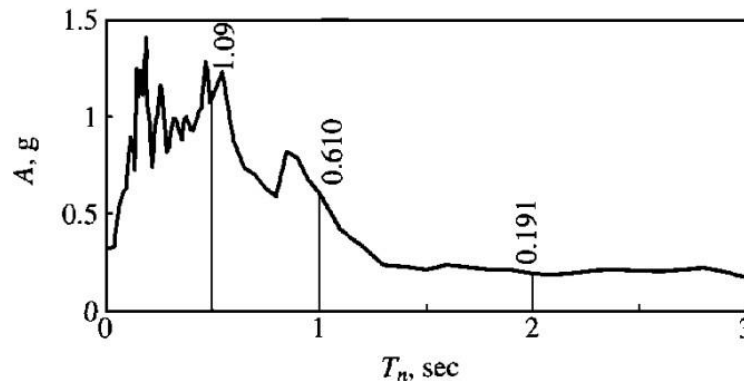


Pseudo-velocity Response Spectrum

$$V = \omega_n D = \frac{2\pi}{T_n} D$$

Gives energy

$$E_{So} = \frac{k u_o^2}{2} = \frac{k D^2}{2} = \frac{k (V/\omega_n)^2}{2} = \frac{m V^2}{2}$$



Pseudo-acceleration Response Spectrum

$$A = \omega_n^2 D = \left(\frac{2\pi}{T_n}\right)^2 D$$

Gives base shear  $V_{bo} = f_{So} = m A = m \omega_n^2 D$

# Combined *D–V–A* Spectrum

Each of the deformation, pseudo-velocity and pseudo acceleration response spectra for a given ground motion contain the same information

The three spectra are simply different ways of presenting the same information on structural response

Displacement response spectra gives max displacement

Velocity response spectra gives max energy stored

Acceleration response spectra gives max equivalent static force/base shear

Three spectra are inter related as follows

$$\frac{A}{\omega_n} = V = \omega_n D \quad \text{or} \quad \frac{T_n}{2\pi} A = V = \frac{2\pi}{T_n} D$$

# 4way log scale paper

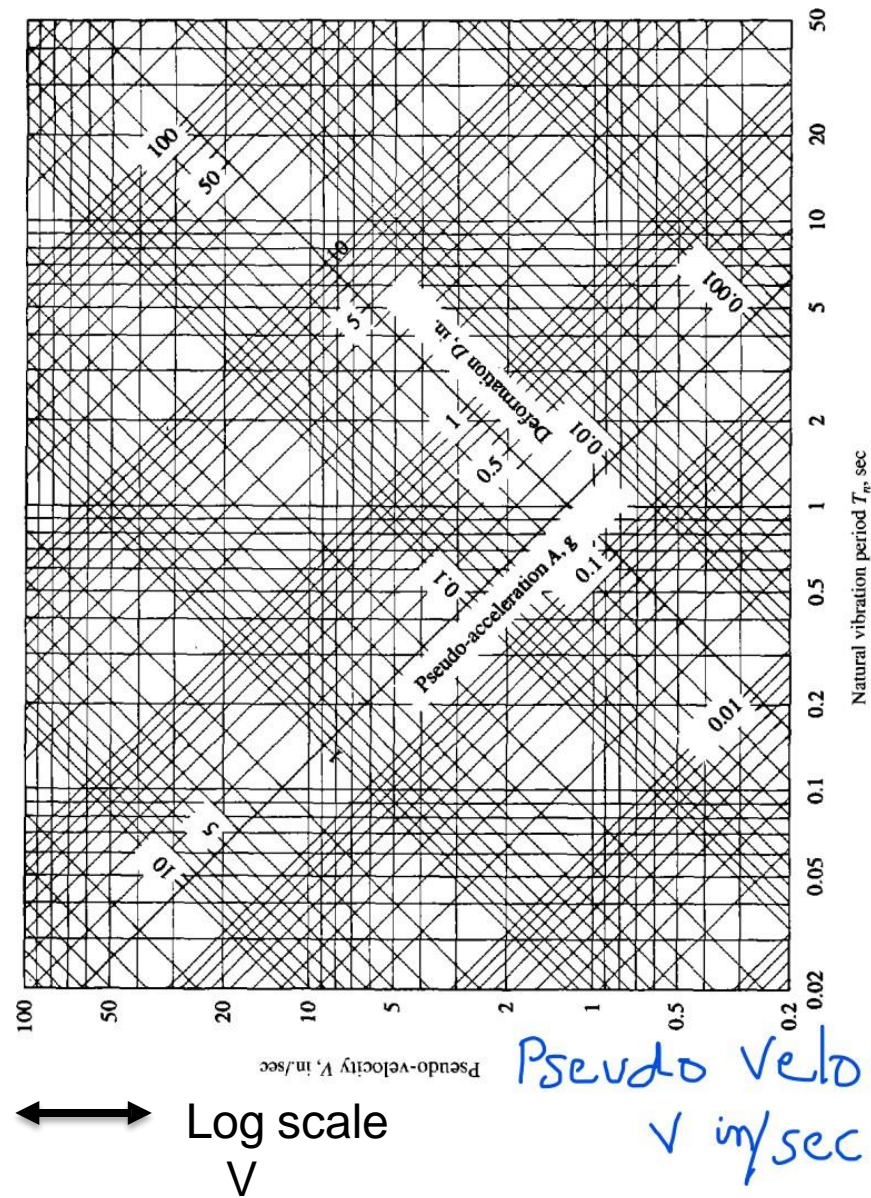


Figure A6.1 Graph paper with four-way logarithmic scales.

Log scale  
 $T_n$



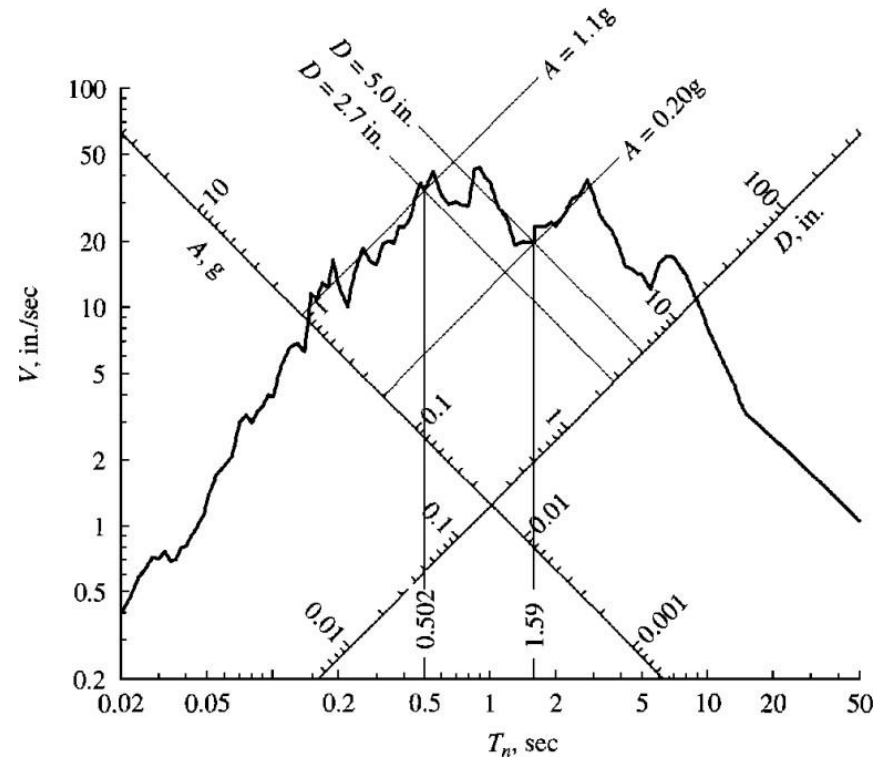
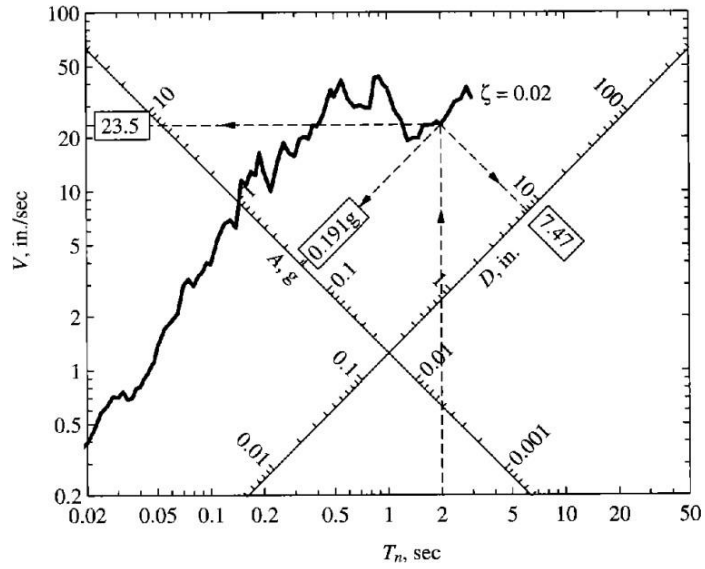
# 3 way response spectra

Three spectra quantities are related to each other as follows

$$\frac{A}{\omega_n} = V = \omega_n D \quad \text{or} \quad \frac{T_n}{2\pi} A = V = \frac{2\pi}{T_n} D$$

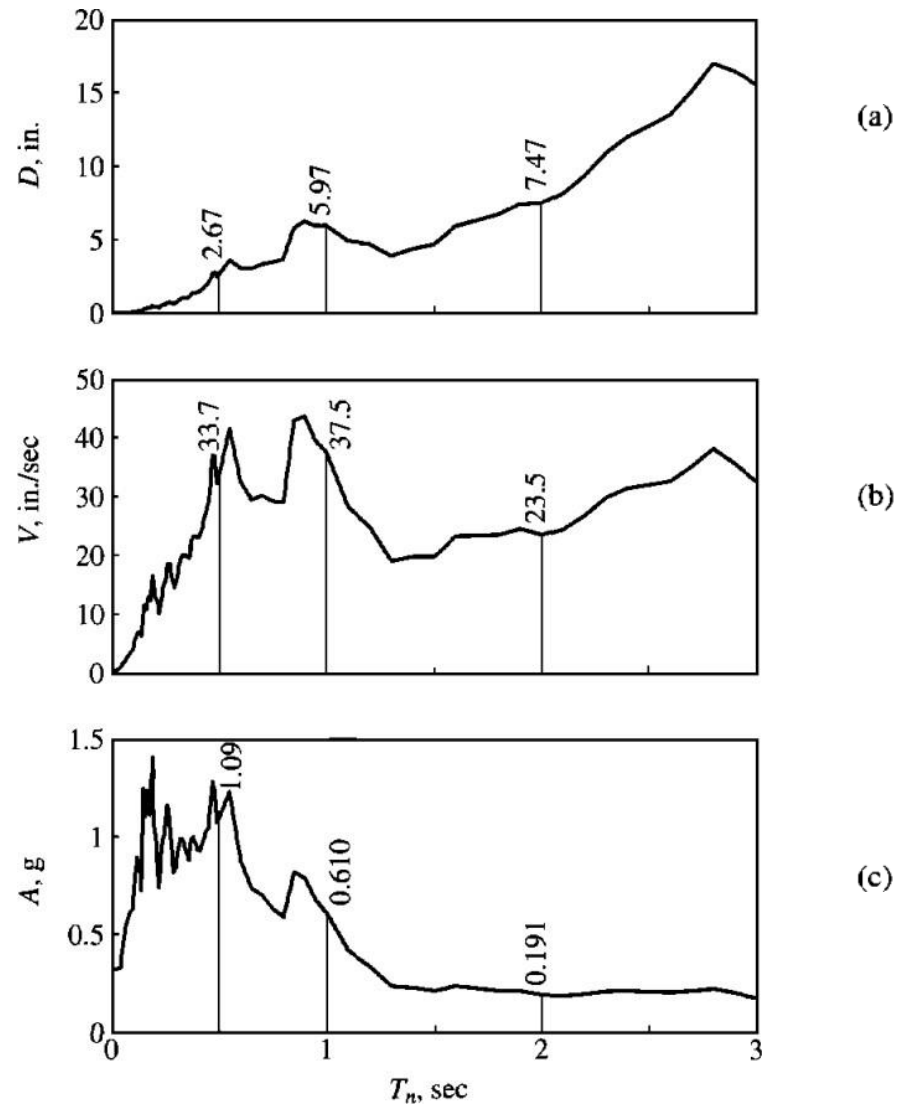


# Combined $D$ - $V$ - $A$ Spectrum

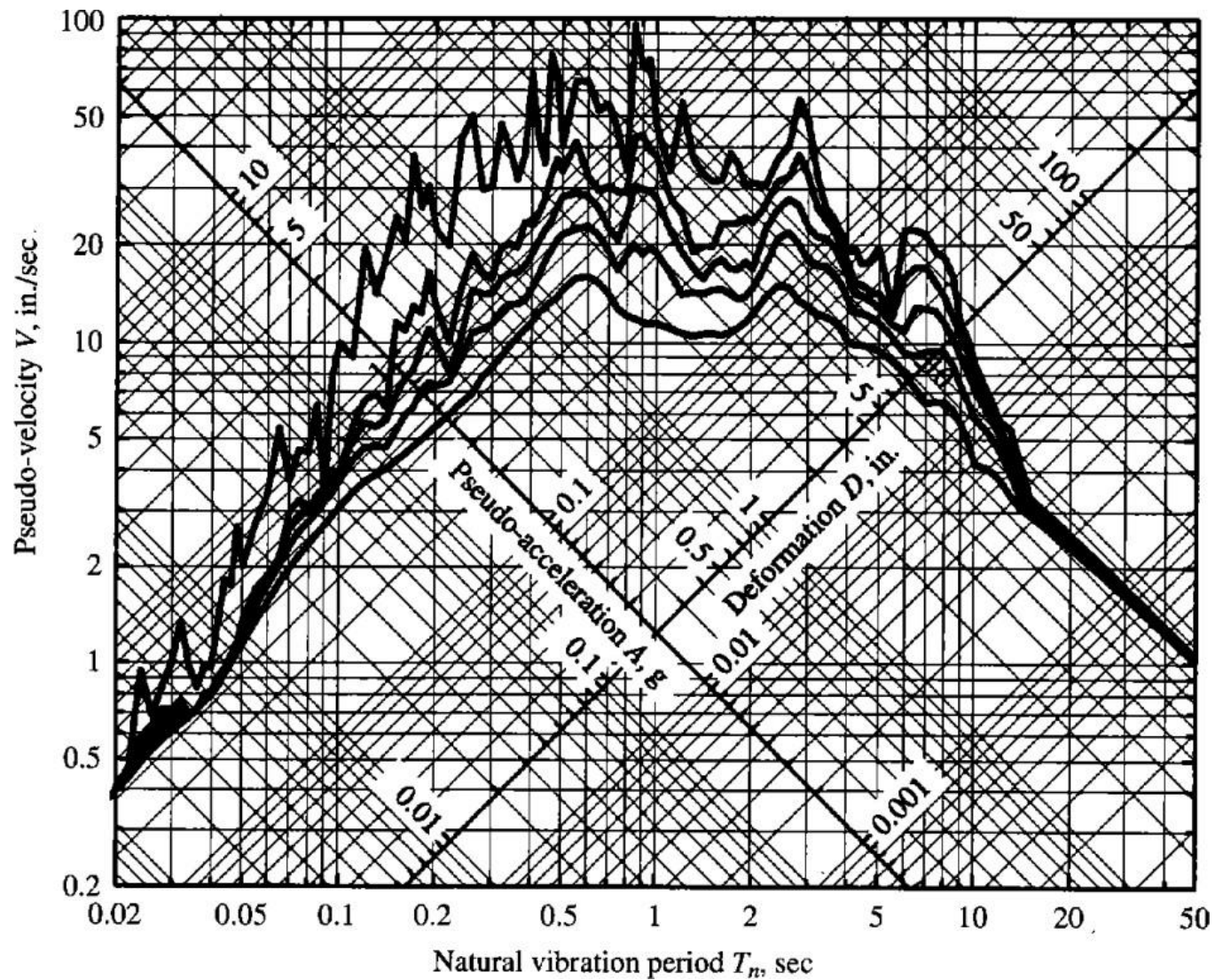


# Construction of Response spectrum

1. Numerically define the ground acceleration  $\ddot{u}_g(t)$ ; typically, the ground motion ordinates are defined every 0.02 sec.
2. Select the natural vibration period  $T_n$  and damping ratio  $\zeta$  of a SDF system.
3. Compute the deformation response  $u(t)$  of this SDF system due to the ground motion  $\ddot{u}_g(t)$  by any of the numerical methods
4. Determine  $u_o$ , the peak value of  $u(t)$ .
5. The spectral ordinates are  $D = u_o$ ,  $V = (2\pi/T_n)D$ , and  $A = (2\pi/T_n)^2 D$ .
6. Repeat steps 2 to 5 for a range of  $T_n$  and  $\zeta$  values covering all possible systems of engineering interest.
7. Present the results of steps 2 to 6 graphically to produce three separate spectra or a combined spectrum



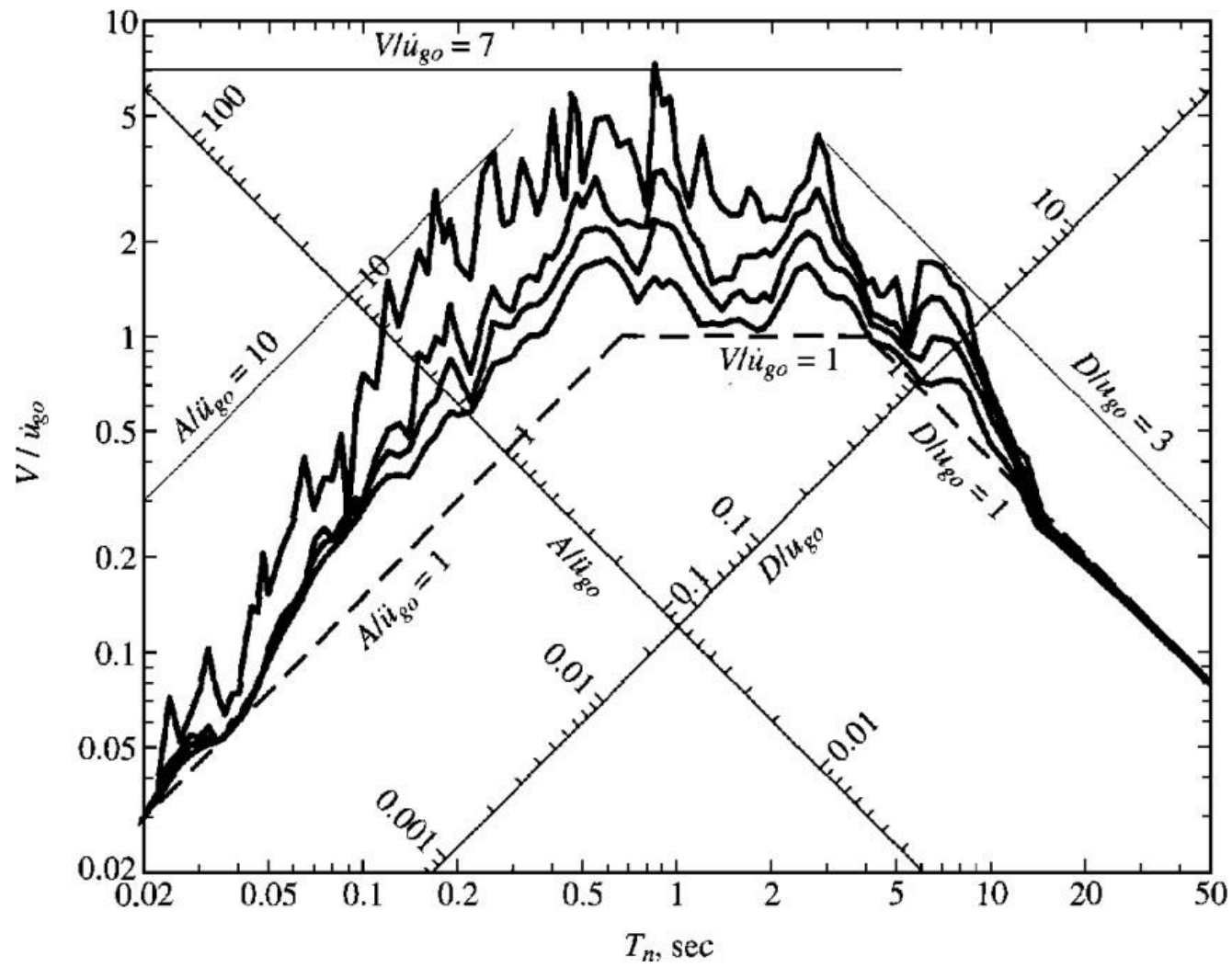
Response spectra ( $\zeta \approx 0.02$ ) for El Centro ground motion: (a) deformation response spectrum; (b) pseudo-velocity response spectrum; (c) pseudo-acceleration response spectrum.



Combined  $D-V-A$  response spectrum for El Centro ground motion;

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 $\zeta = 0, 2, 5, 10, \text{ and } 20\%$ .

# Normalized Response spectrum

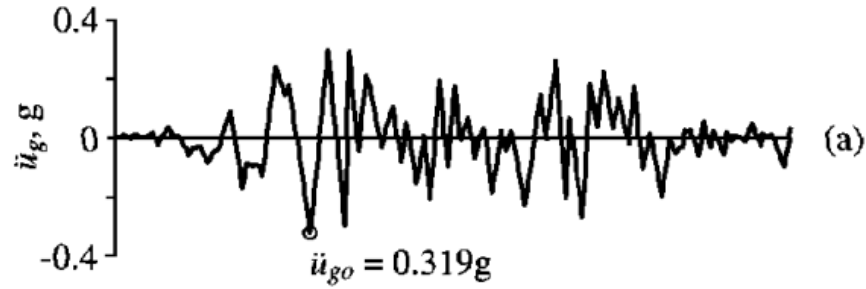


Response spectrum for El Centro ground motion plotted with normalized scales  $A/\ddot{u}_{go}$ ,  $V/\ddot{u}_{go}$ , and  $D/\dot{u}_{go}$ ;  $\zeta = 0, 2, 5, \text{ and } 10\%$ .

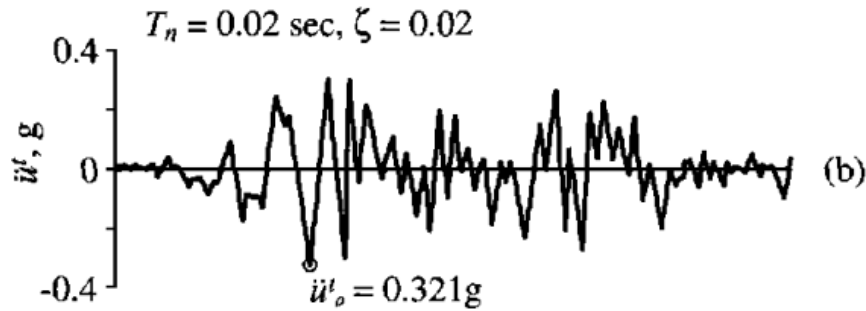
# Response of a very rigid system $T = 0.02$

For this system, the structure is very rigid and hence the mass acceleration will be same as ground acceleration

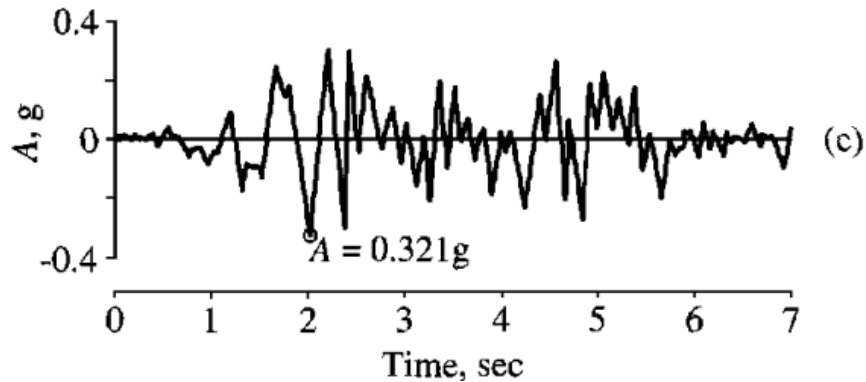
# Response of a very rigid system $T = 0.02$



El Centro ground acceleration;



total acceleration response of an SDF system  
with  $T_n = 0.02$  sec and  $\zeta = 2\%$ ;

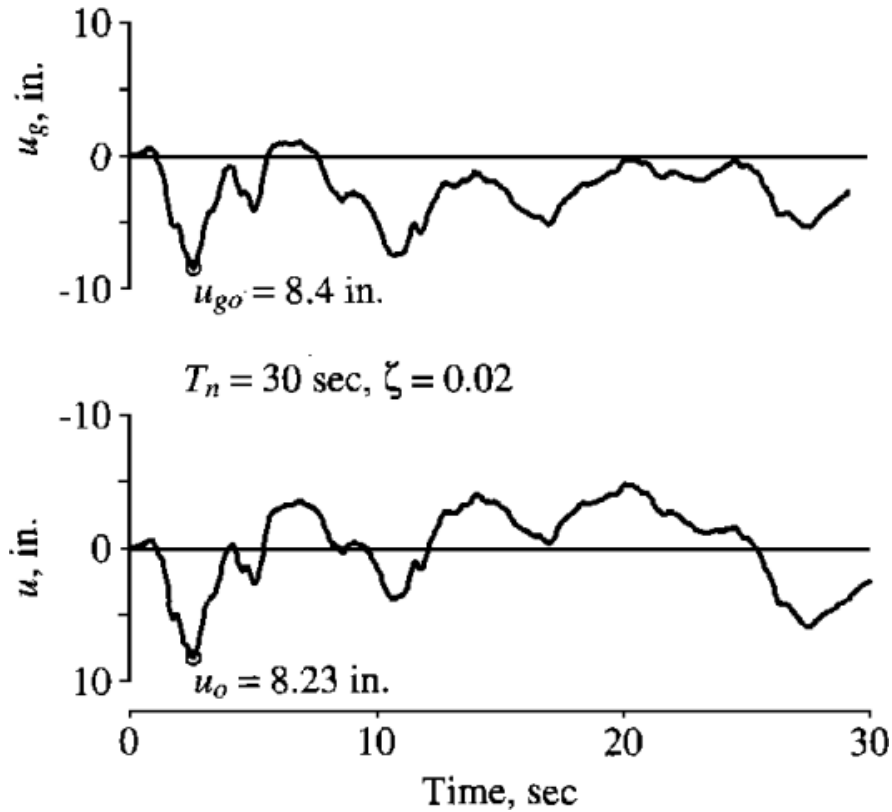


pseudo-acceleration response of the same system;



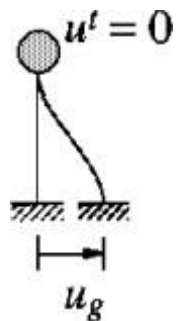
rigid system.

# Response of a very Flexible system $T = 30$ sec



El Centro ground displacement;

deformation response of SDF system  
with  $T_n = 30$  sec and  $\zeta = 2\%$



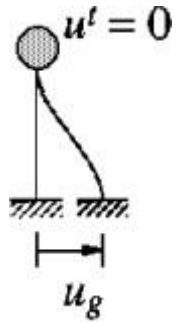
very flexible system.



# Response of a very flexible system $T > 15$ sec

For this system, structure is very flexible, and hence the mass would remain stationary resulting into

$$u(t) \simeq -u_g(t)$$



# Response of short period system

## $0.035 < T < 0.5 \text{ sec}$

For short-period systems with  $T_n$  between  $T_a = 0.035 \text{ sec}$  and  $T_c = 0.50 \text{ sec}$

Acceleration of mass exceeds ground acceleration and  
Magnification depends on  $T_n$  and damping

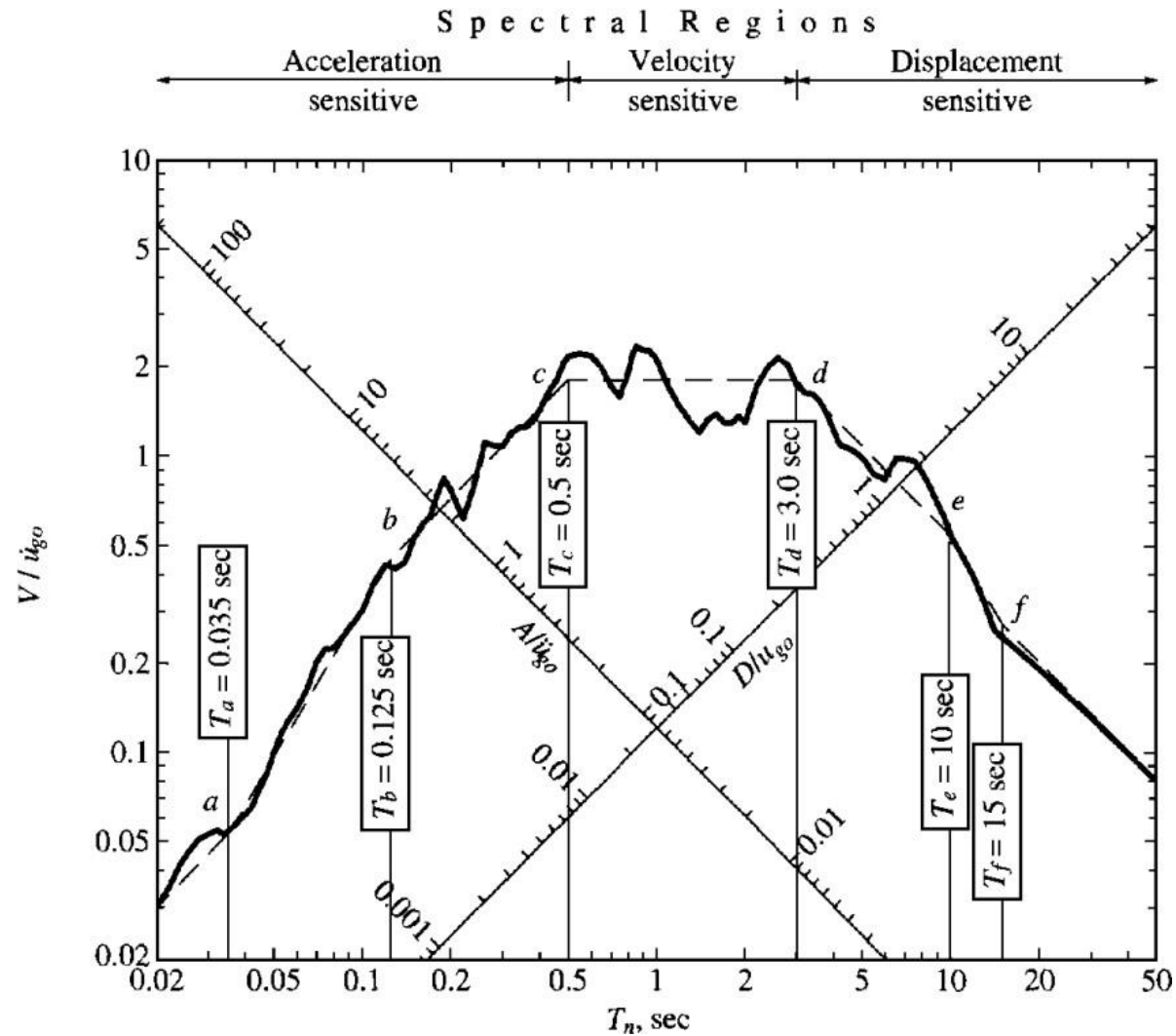
For  $0.125 < T_n < 0.5$  the mass **acceleration** is constant equal to ground **acceleration** magnified by a factor depending on damping

For  $0.5 < T_n < 0.3$  the mass **velocity** is constant equal to ground **velocity** magnified by a factor depending on damping

For  $3 < T_n < 15$  the mass **Displacement** is greater than ground **Displacement** and magnification depends on  $T_n$  and damping

For  $3 < T_n < 10$  the mass **Displacement** is constant equal to ground **Displacement** magnified by a factor depending on damping

# Spectrum is divided into three zones



Solid line – response spectrum for El centro earthquake

Dashed line – idealized response spectrum for El centro earthquake

# Elastic Design spectrum

Generally response spectrum of each recorded earthquake ground motion is different

Elastic design spectrum is used to design new structures for future earthquake

The Design spectrum should be representative of ground motion recorded at site during past earthquake

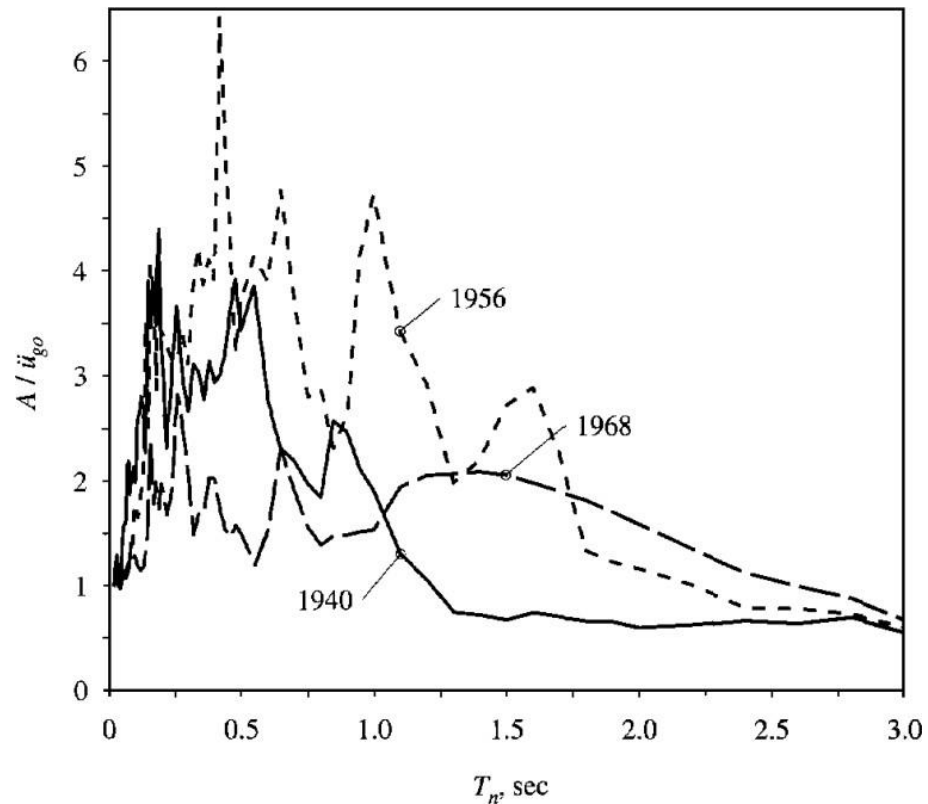
If such record is not available, the design spectrum should be based on The record

If such record is not available, the design spectrum should be based on the record available at other site under similar condition

The factors to be matched are

1. Magnitude of earthquake
2. Distance of site from source of earthquake
3. Fault mechanism
4. Geology of travel path
5. Local soil conditions

If such records in sufficient numbers are not available then statistical approach is necessary to consider available records and do some averaging of results

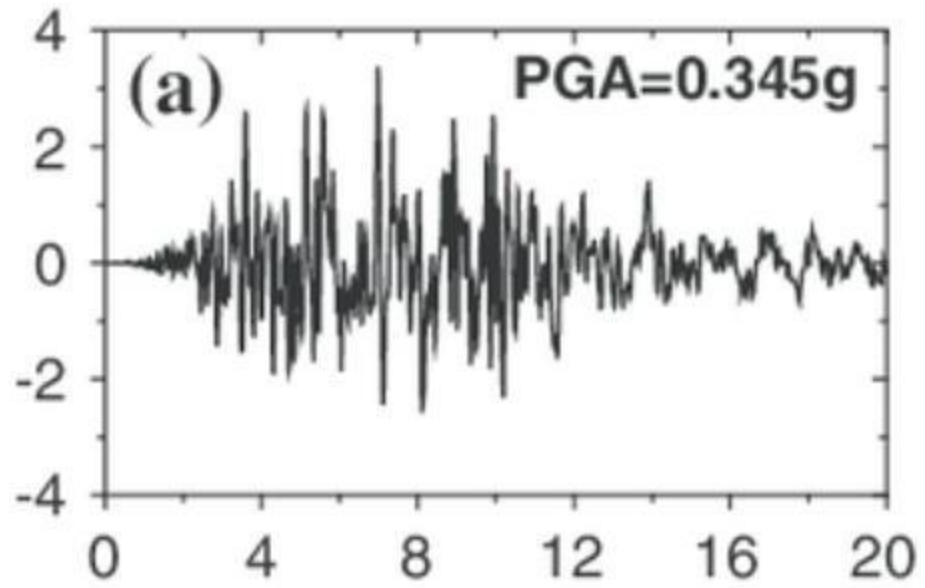


Response spectra of Imperial valley earthquakes, El centro California  
18 may 1940  
9 February 1956  
8 April 1968

Y axis is normalized mass acceleration  
= mass acceleration/ground acceleration

Typical Seismic ground motion

Ground Acceleration ( $\text{m/s}^2$ )

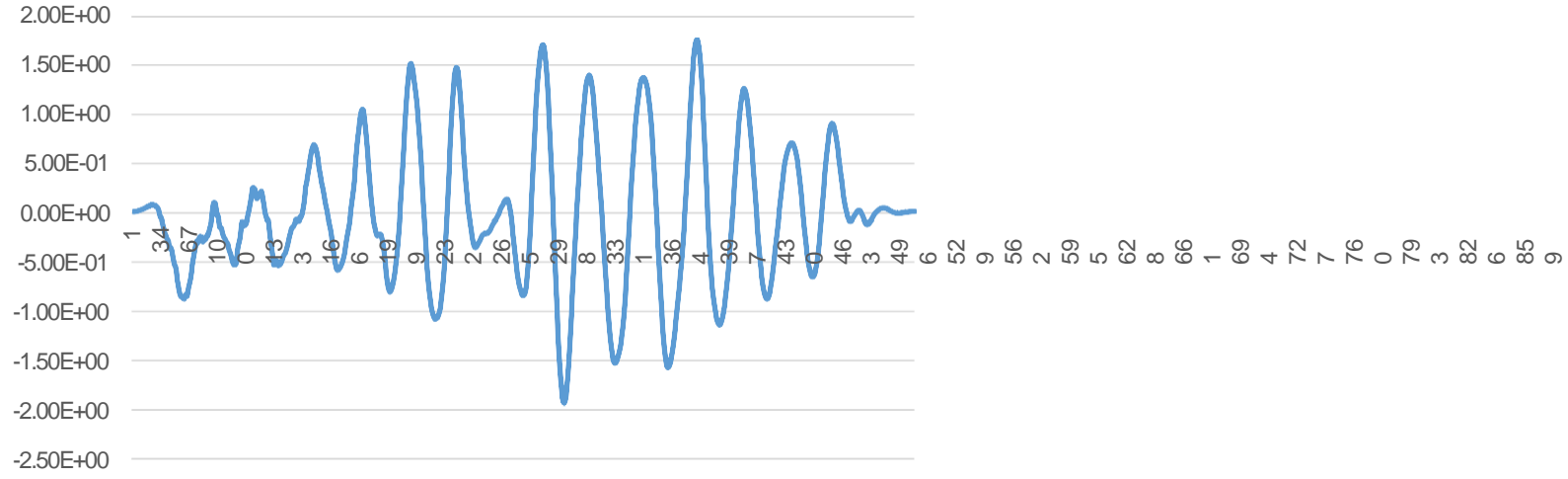


# Time history analysis using ETABS and Response spectra

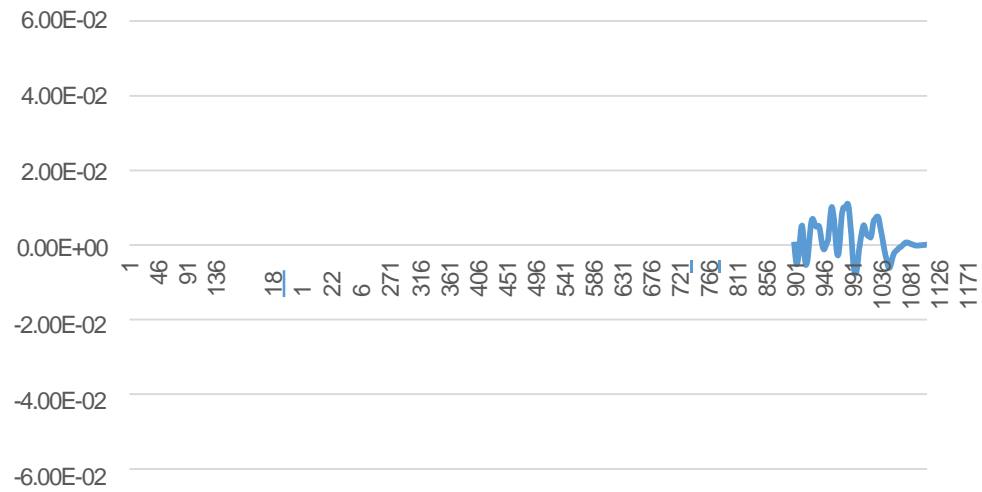
M. G. Gadgil



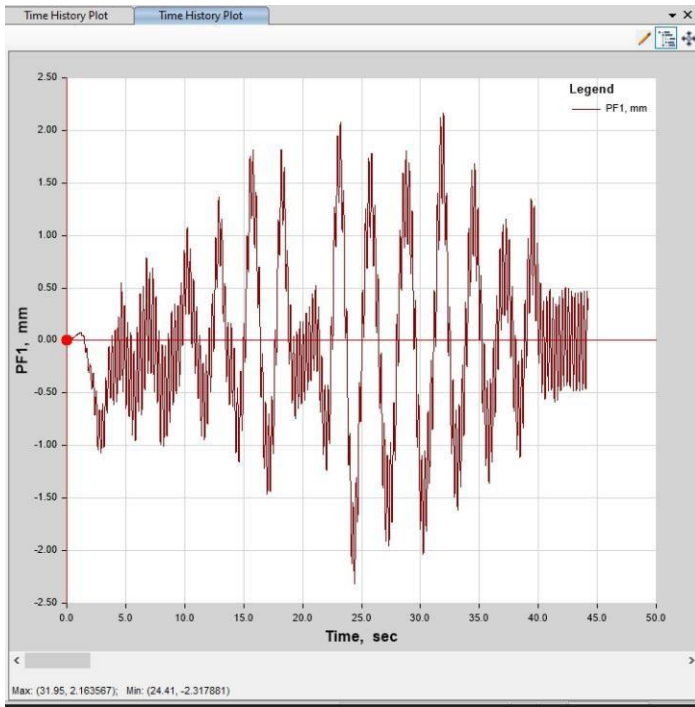
### park field



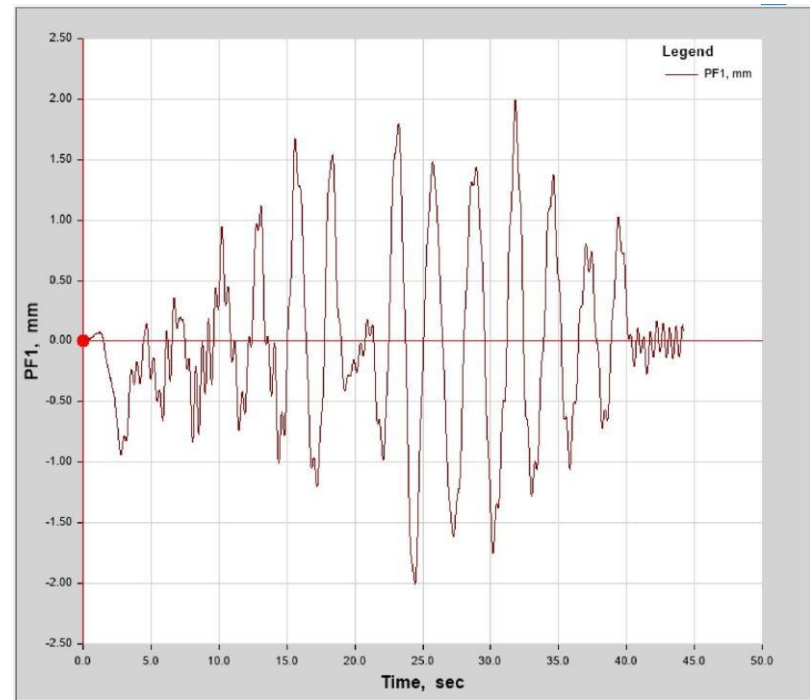
### Imperial valley



# Time history analysis for Parkfield earthquake ground motion

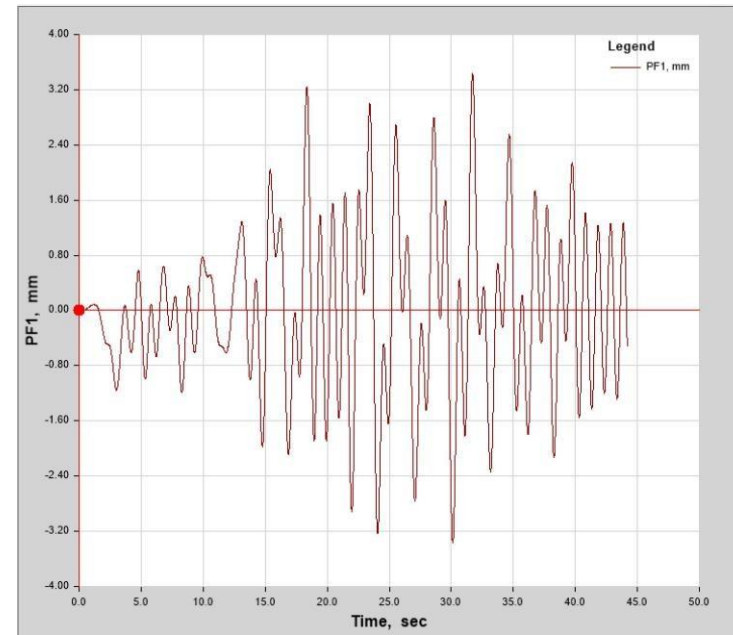
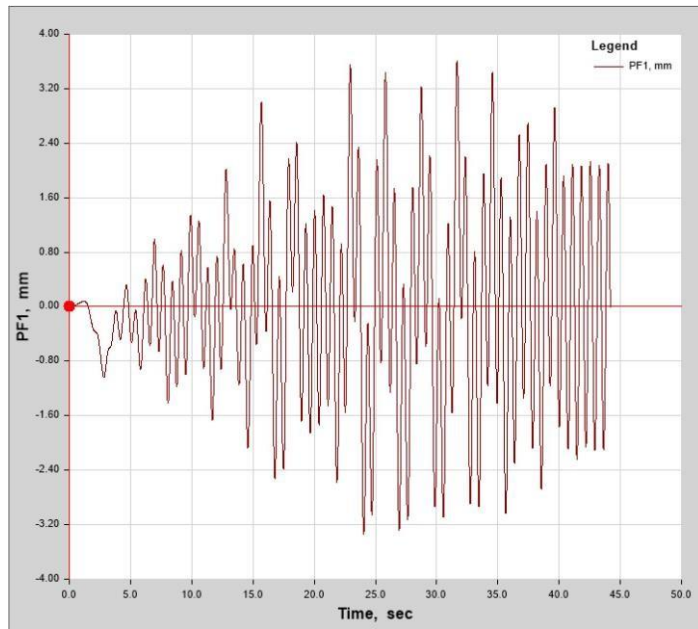


$T = 0.263$  sec  
 $D_{max} = 2.31$



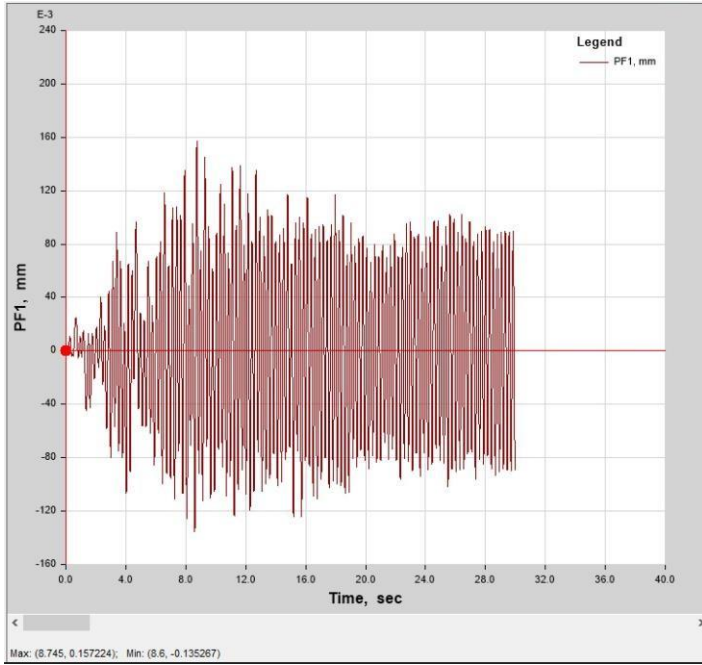
$T = 0.4739$  sec  
 $D_{max} = 2$  mm

$T = 0.726$  sec  
 $D_{max} = 3.61$  mm

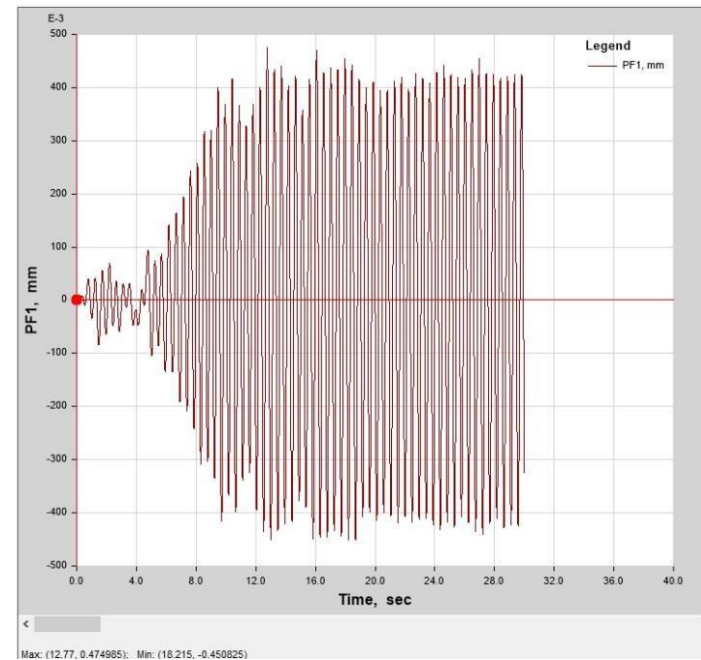


$T = 1.016$  sec  
 $D_{max} = 3.42$  mm

# Time history analysis for Imperial valley earthquake ground motion

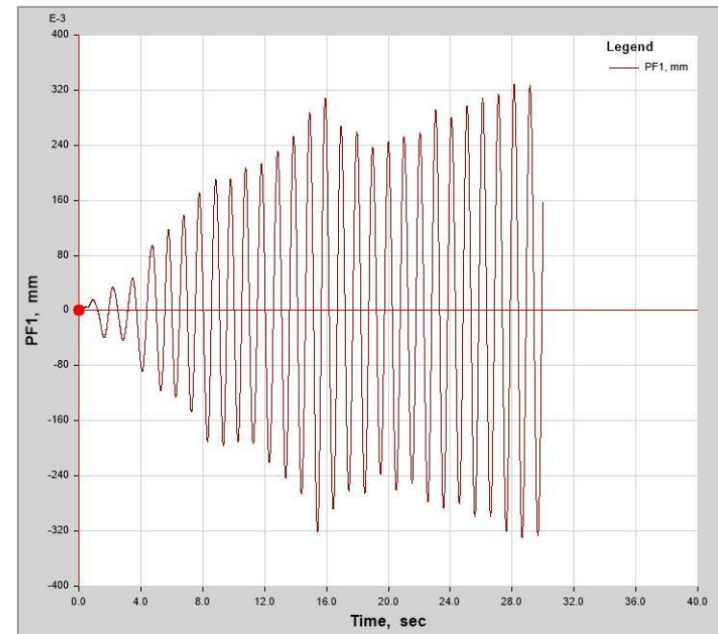
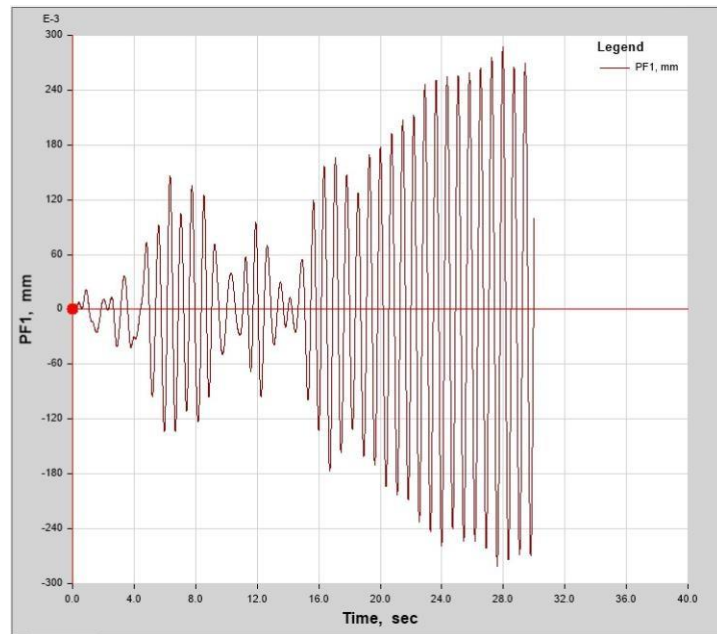


$T = 0.263 \text{ sec}$   
 $D_{\text{max}} = 0.15$



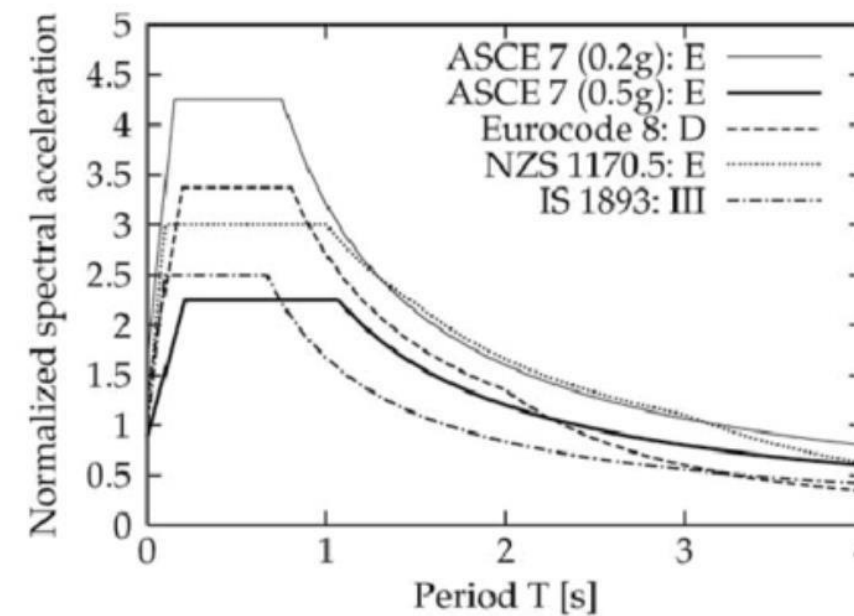
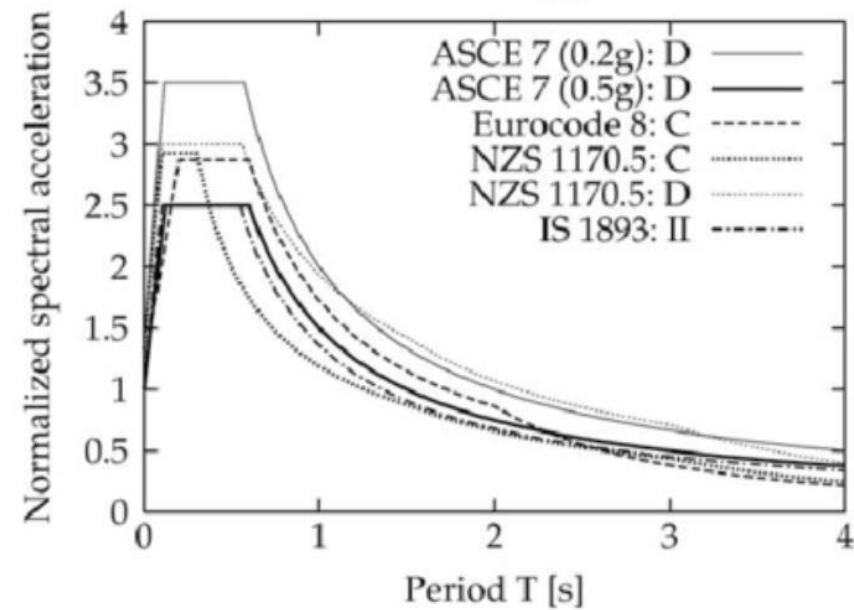
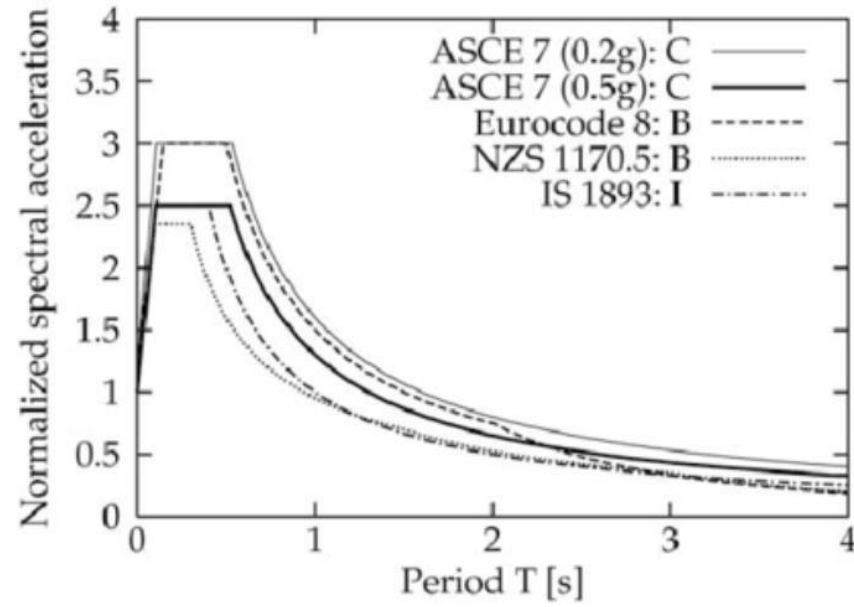
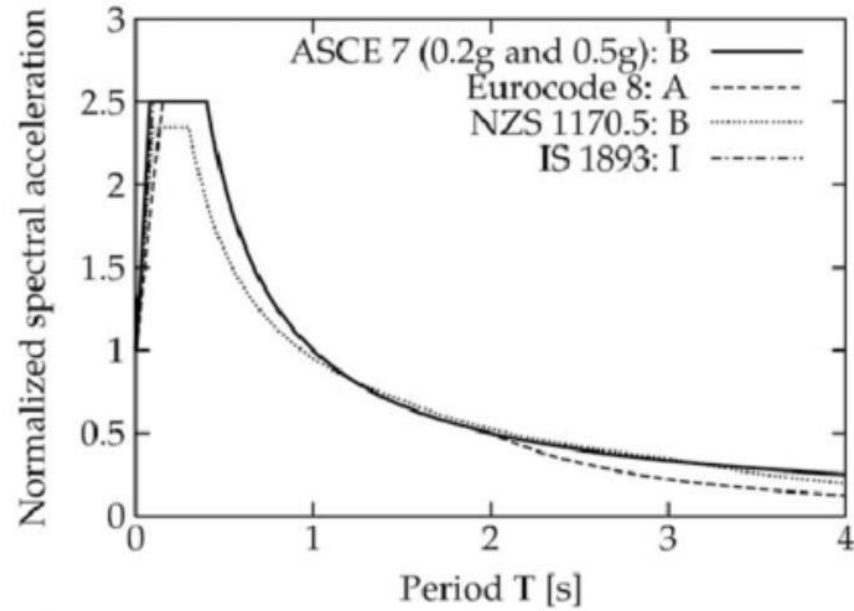
$T = 0.4739 \text{ sec}$   
 $D_{\text{max}} = 0.47 \text{ m}$

$T = 0.726 \text{ sec}$   
 $D_{\text{max}} = 0.28 \text{ mm}$



$T = 1.016 \text{ sec}$   
 $D_{\text{max}} = 3.42 \text{ mm}$

# Response spectra of different countries



IS : 1893 - 1975

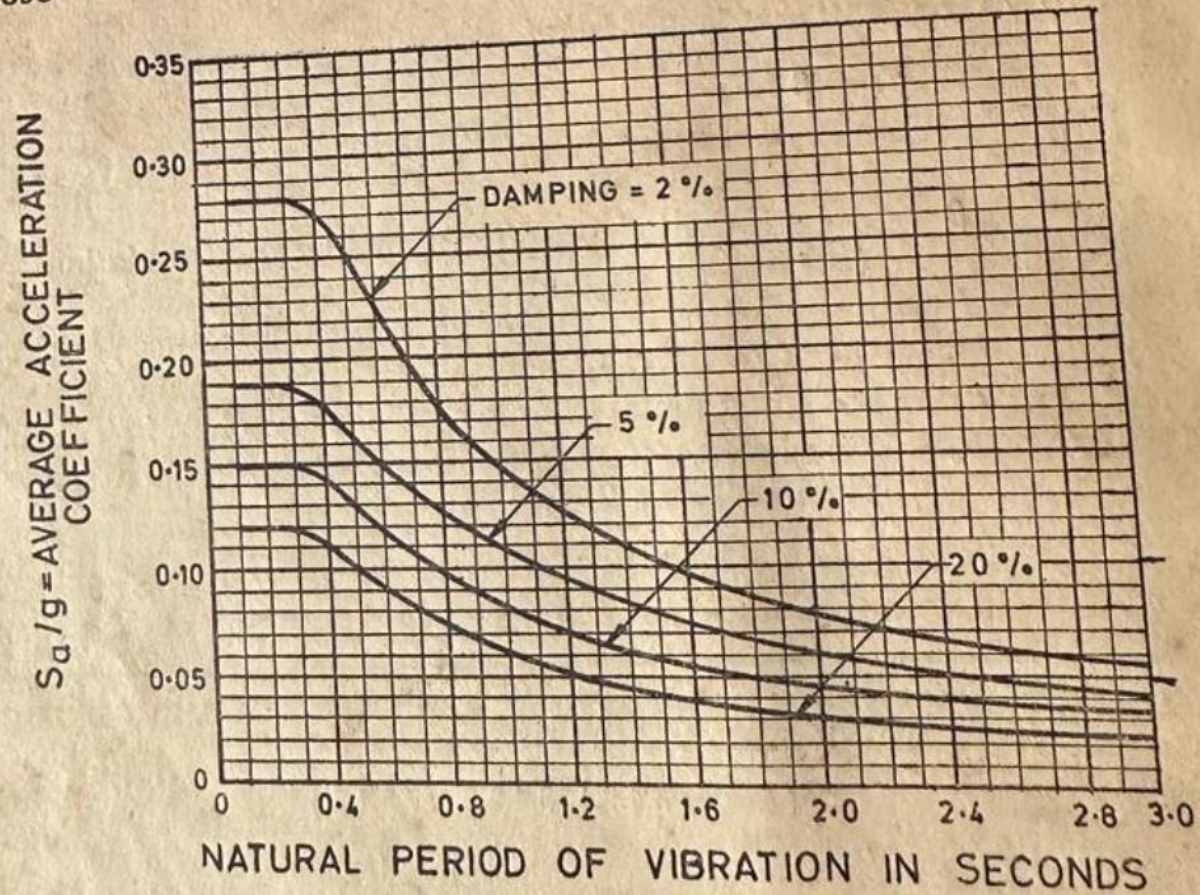


FIG. 2 AVERAGE ACCELERATION SPECTRA

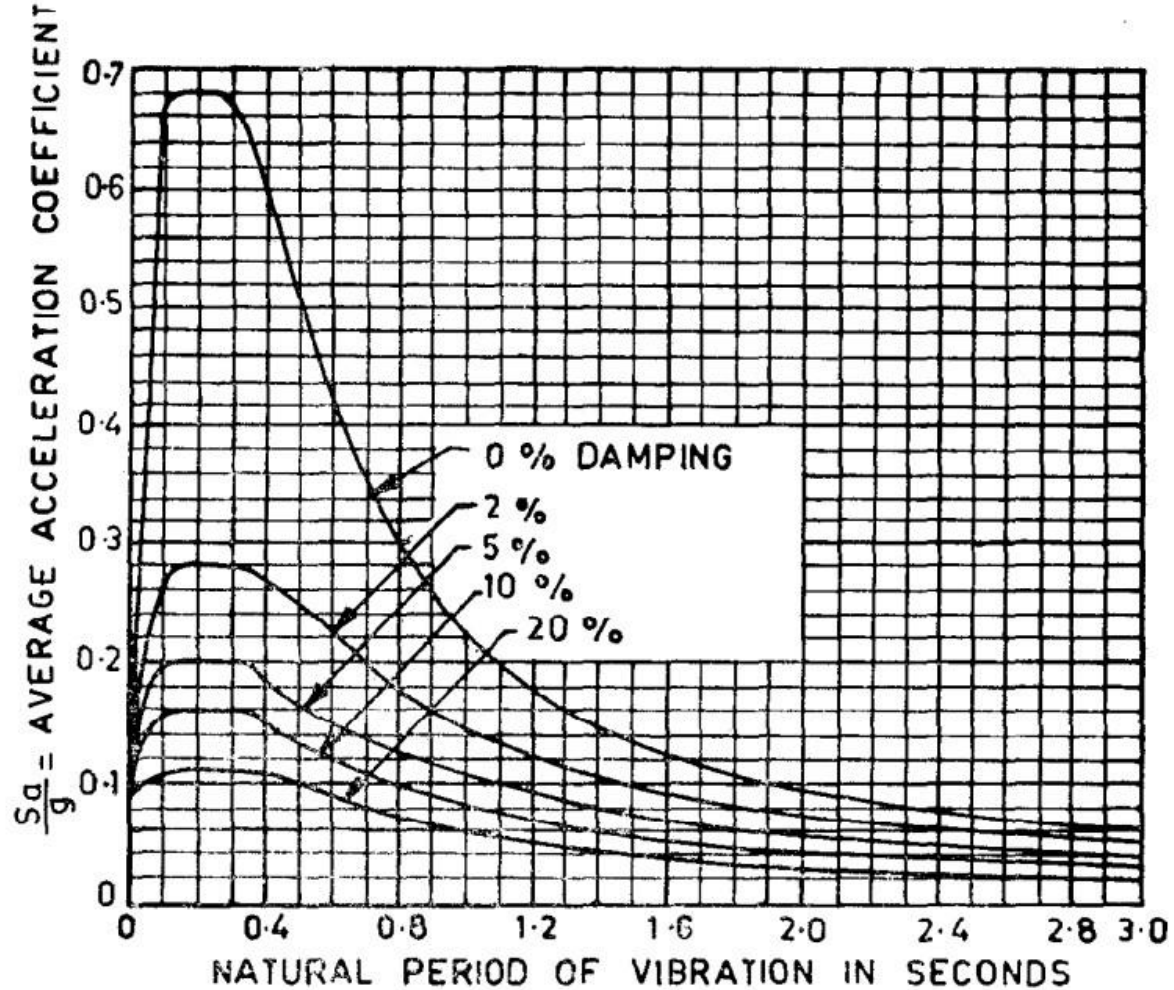


FIG. 2 AVERAGE ACCELERATION SPECTRA

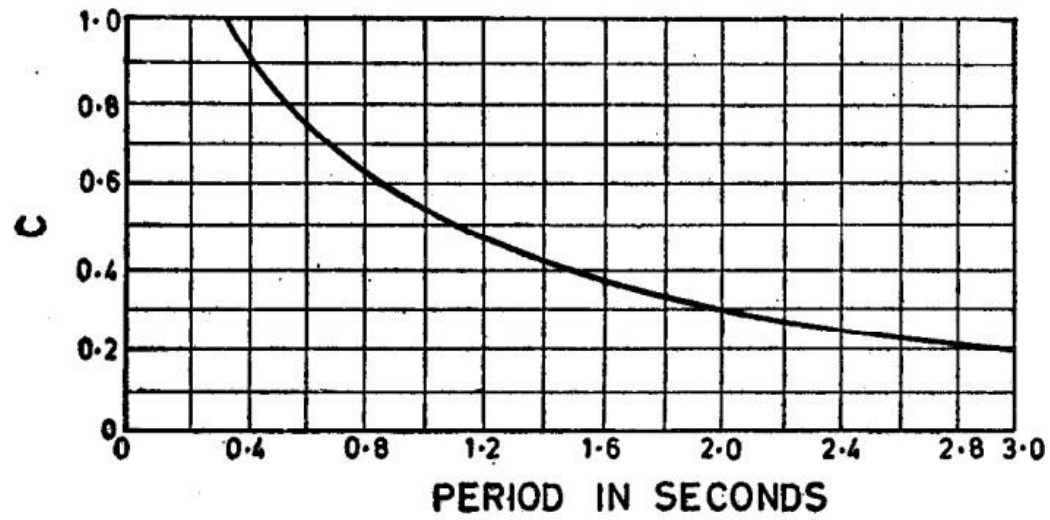


FIG. 3 C Versus PERIOD

# IS 1893 ( Part 1 ) : 2002

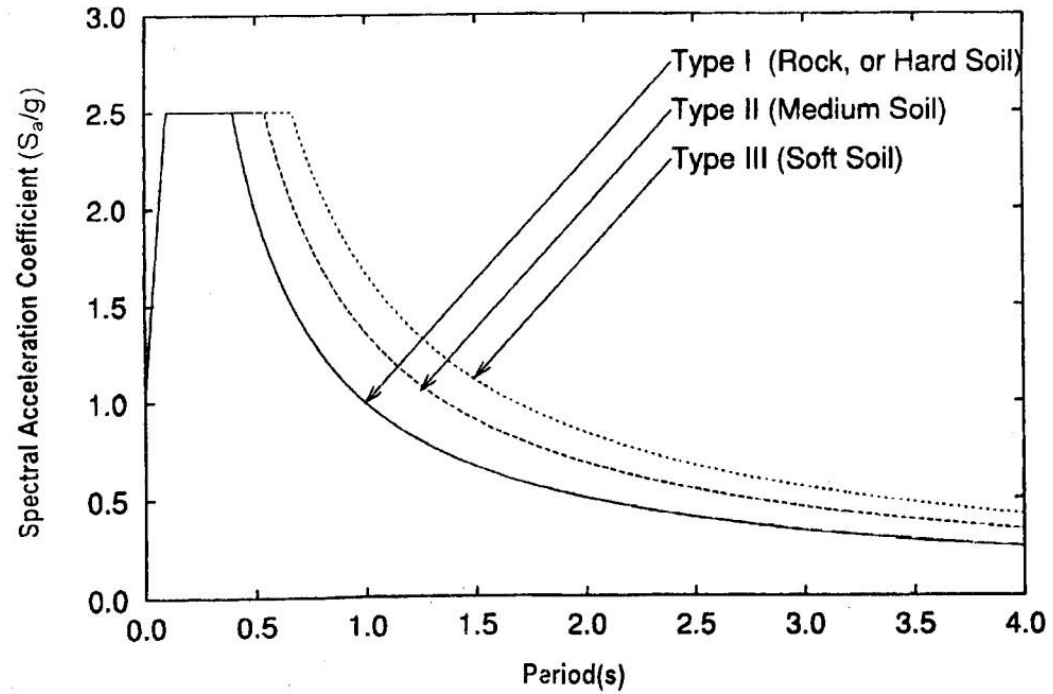
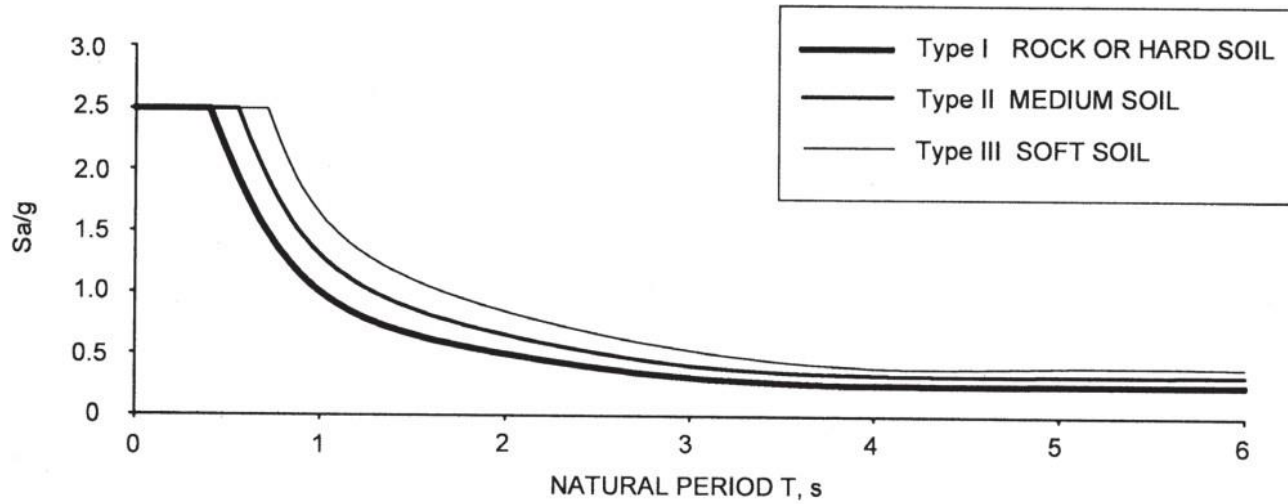
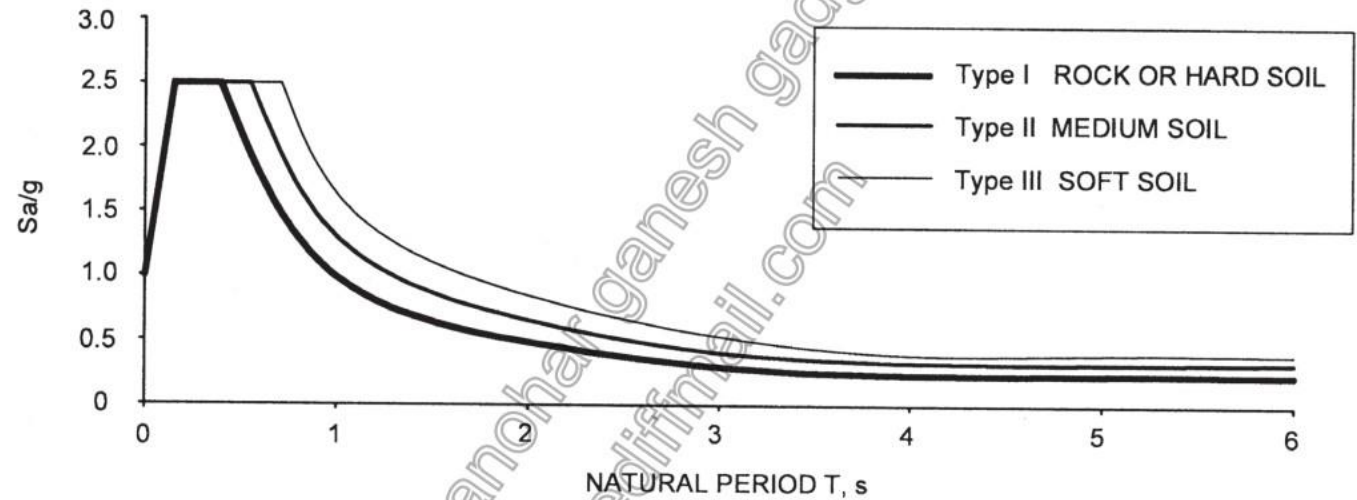


FIG. 2 RESPONSE SPECTRA FOR ROCK AND SOIL SITES FOR 5 PERCENT DAMPING

# IS 1893 (Part 1) : 2016



2A SPECTRA FOR EQUIVALENT STATIC METHOD



2B SPECTRA FOR RESPONSE SPECTRUM METHOD