WELCOME

20-May -2023

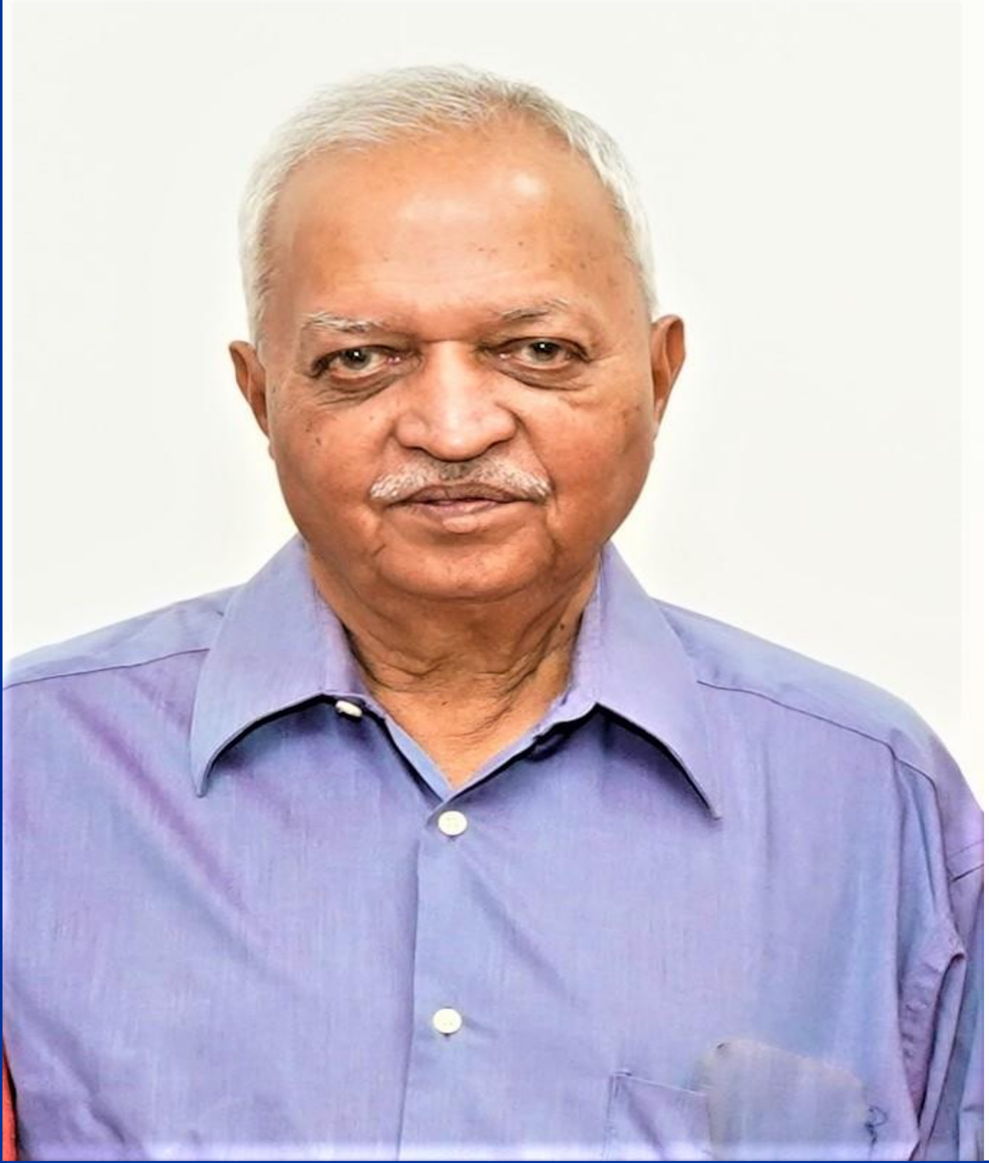
Session 7

Structural Engineering
Overview and basic concepts of Dynamics

By

Prof. M. G. Gadgil

Prof. Manohar G Gadgil



Prof. Manohar Gadgil is retired professor from VJTI. He was HOD of the structural department of VJTI.

He completed his Bachelor of Engineering in Civil from the University of Bombay in 1970 and M. Tech. in Structure from I.I.T. Powai in 1975.

He has published several papers at Indian and international conferences.

During the last 33 years, he has guided more than 100 P.G. students in their dissertation work.

The software needed for the projects was developed by him in the days when ready-to-use software was not available on the market.

He consults on several industry-sponsored projects like high-rise buildings, machine foundation equipment, industrial building structures, and many more.

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Primary objective of structural engg activity

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Primary objective of structural engg activity

To plan, design and detail a structural system which will carry safely the loads coming on it.

i. **We have a structural system**
ii. **It has to carry loads**
iii. **The loads be carried safely and efficiently**

Basic classification of structures

Structures

Structures as understood by layman

i. Buildings
ii. Bridges
iii. Fluid container tanks
iv. Retaining walls
v. Dams
vi. Aircrafts
vii. Ships
viii. Automobiles
ix. Tunnels
x. Jetty
xi. Cranes
xii. Balloon

Classifica

i. Beam
ii. Column
iii. Arch
iv. Cable
v. Truss--- plane/space
vi. Frame– plane/space
vii. Floor grid
viii. Slab
ix. Shell
x. Folded plate
xi. 3-d solid
xii. Plane stress
xiii. Plane strain
xiv. Axisymmetric problems

We have a structure

Def. Structure is an assemblage of members/elements suitably connected to each other and supported so that it carries loads coming on it safely to the support/foundation

Thus the basic building blocks of any structure is a member or an element which may be one, two or three dimensional

Building blocks of structure

Basic building blocks of structures forces

i. Truss element--- AF
ii. Beam element --- SF and BM
iii. Frame element 2-3 d--- AF, SF, BM
iv. Cable element 2d AF tension
v. Arch element 2d AF, SF, BM
vi. Slab element/Grid--- BM, TM, SF
vii. Plane stress element in- $\sigma_x \sigma_y$ τ_{xy}
viii. Plane strain element– in plane stresses and out of plane stress $\sigma_x \sigma_y \sigma_z \tau_{xy}$
ix. Membrane element--- $\sigma_x \sigma_y$
x. Shell element -- $N_x N_y N_{xy} Q_x Q_y M_x M_y M_{xy}$
xi. Solid element $\sigma_x \sigma_y \sigma_z$

Basic building blocks of structures displacements

i. Truss element--- δx, δy
ii. Beam element --- θz, δy
iii. Frame element 2-3 d--- δx, δy, δz, θx, θy, θz
iv. Slab element--- θx, θy, δz
v. Plane stress element in- δx, δy
vi. Plane strain element– δx, δy
vii. Membrane element--- u,v,w
viii. Shell element – u,v,w, θx, θy, θz
ix. Solid element δx, δy, δz

Internal forces in members of a structure

i. Axial force

ii. Shear force

iii. Bending moment

iv. Torsion

Internal stresses in any members

i. Normal stress tensile
ii. Normal stress compressive
iii. Shear stress

Materials

Materials of structure

I. Natural materials
II. Man made materials
III. Combination materials
IV. Structural materials
V. Aesthetic materials
VI. Functional materials

Properties of material required in structural analysis and design

i. Density
ii. Elastic modulus
iii. Shear modulus
iv. Bulk modulus
v. Ultimate stress in Tension /compression/ shear
vi. Poisson’s ratio
vii. Percentage elongation
viii. Yield stress/proof stress
ix. Coefficient of thermal expansion

Material properties-- Physical

Strength

Elasticity

Plasticity

Hardness

Toughness

Brittleness

Stiffness

Ductility

Malleability

Cohesion

Impact strength

Fatigue

Creep

Assumptions made regarding any material used in structure

i. Material is isotropic
ii. Material is homogeneous
iii. Material is linearly elastic

Loads on structure

Loads on structure

i. Self wt
ii. Imposed load --- floor finish, live load
iii. Wind load
iv. Seismic load
v. Temperature loads
vi. Shrinkage and creep
vii. Fatigue
viii. Lack of fit
ix. Pre-stress
x. Buoyancy
xi. Pressure(air/liquid/soil)
xii. Electro-Magnetic force
xiii. Settlement of support

Static v/s Dynamic Loads

- Static Loads
- Forces are constant
- Response is constant wrt time
- Example Self Wt, gradually applied Live load, normal wind load

- Dynamic Loads
- Force varies for it's magnitude/Direction/ or Point of application
- Support Motion
- Response is variable wrt time
- Example high speed machine induced periodic force, explosion, Impact, moving load on bridges, Ground motion during earthquake

Static v/s Dynamic Response

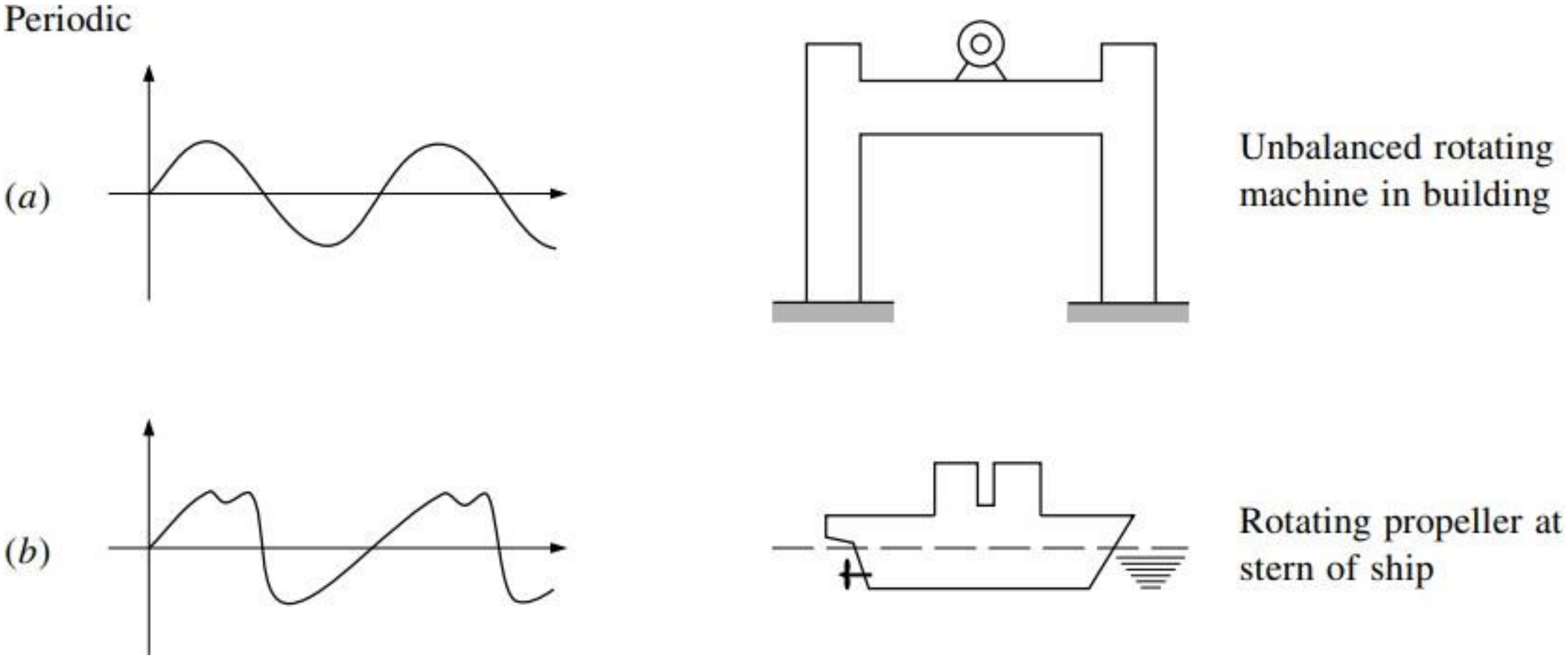
- Static Response is a function of
- Type of force
- Stiffness of structure
- Static response = Force/stiffness

- Dynamic Response is a function of
- Type of force
- Stiffness of structure
- Mass of structure
- Dynamic response = static response * dynamic magnification factor

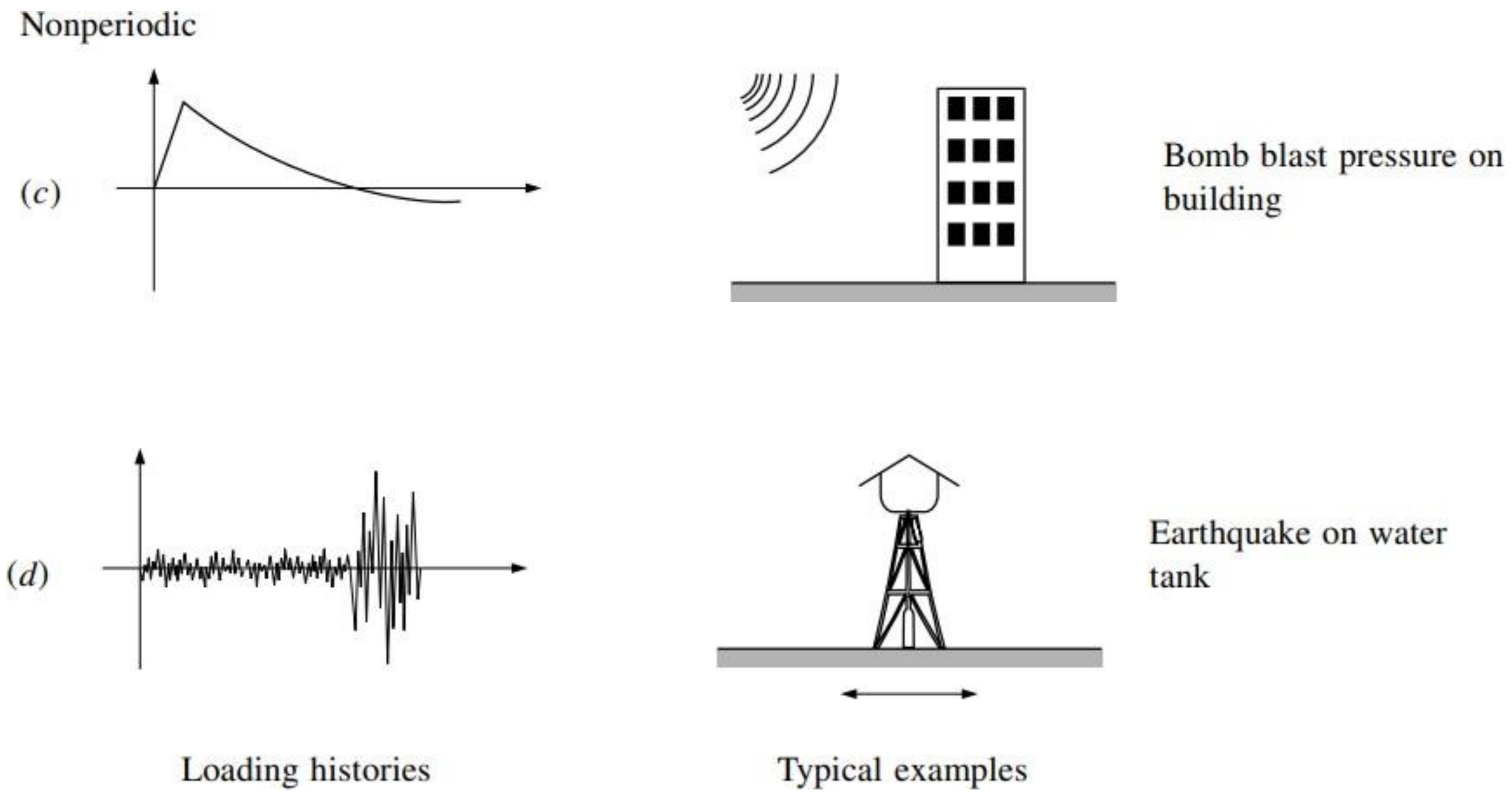
Dynamic Magnification Factor

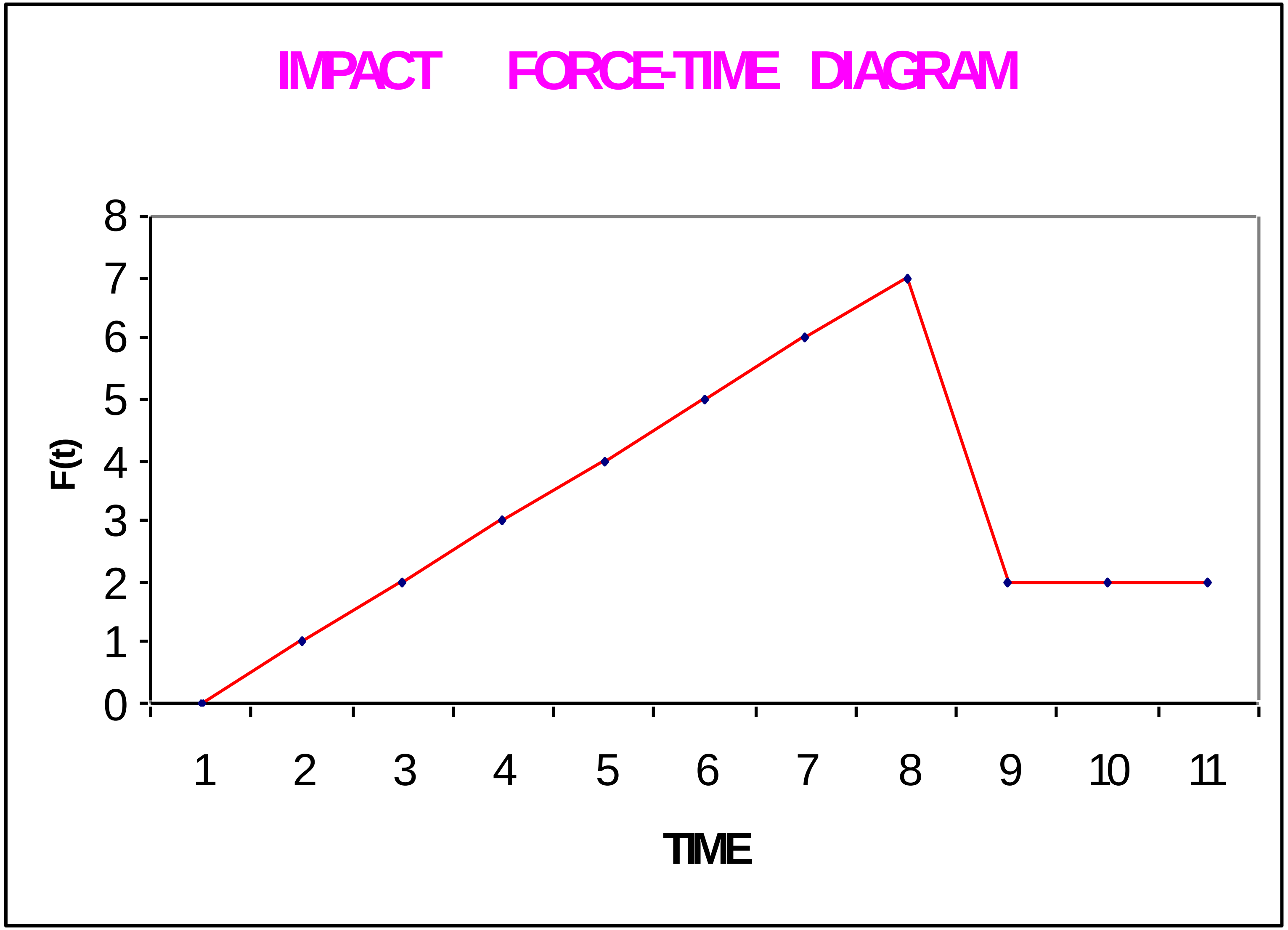
- It is a Function of
- Type of Dynamic force i.e. force-time diagram of the given force-- Given
- Duration of Dynamic force - Given
- Period of Natural vibration of the structure, denoted as T– a Function of structure stiffness and Mass

Various types of dynamic forces

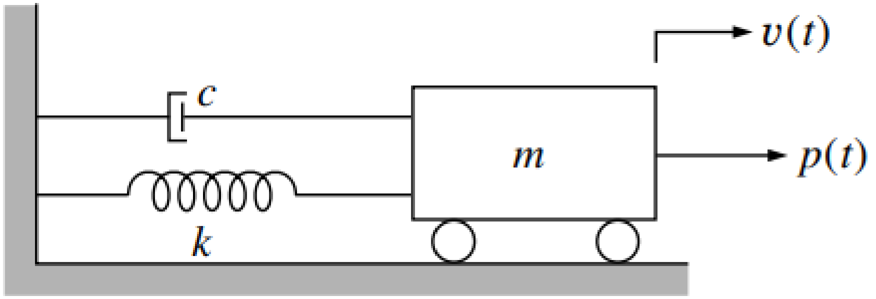


Dynamic forces contd

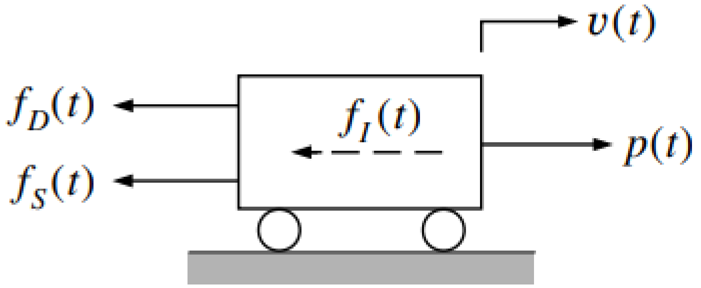



IMPACT FORCE-TIME DIAGRAM
F(t)
0
1
2
3
4
5
6
7
8
1
2
3
4
5
6
7
8
9
10
11
TIME

Basic physical system



Free body diagram
(forces in equilibrium)



equation of motion

$$f_I(t) + f_D(t) + f_S(t) = p(t)$$

$$f_I(t) = m\,\ddot{v}(t)$$ Inertia force

$$f_D(t) = c\,\dot{v}(t)$$ Damping force

$$f_S(t) = k\,v(t)$$ Elastic force

$$m\,\ddot{v}(t) + c\,\dot{v}(t) + k\,v(t) = p(t)$$ Equation of motion

Solution of this equation is obtained in two parts

Complimentary function + Particular integral

Complimentary solution of the equation is obtained by setting RHS to zero

$$m\,\ddot{v}(t) + c\,\dot{v}(t) + k\,v(t) = 0$$

Dividing by m and substituting $\omega^2 = K/M$

Solution of this equation is obtained by taking a function

$$v(t) = A\cos\omega t + B\sin\omega t$$

Substituting this function in above differential equation and taking undamped state (c = 0) we get following solution of the equation

$$v(t) = v(0)\cos\omega t + \dot{v}(0)\,\omega\,\sin\omega t$$

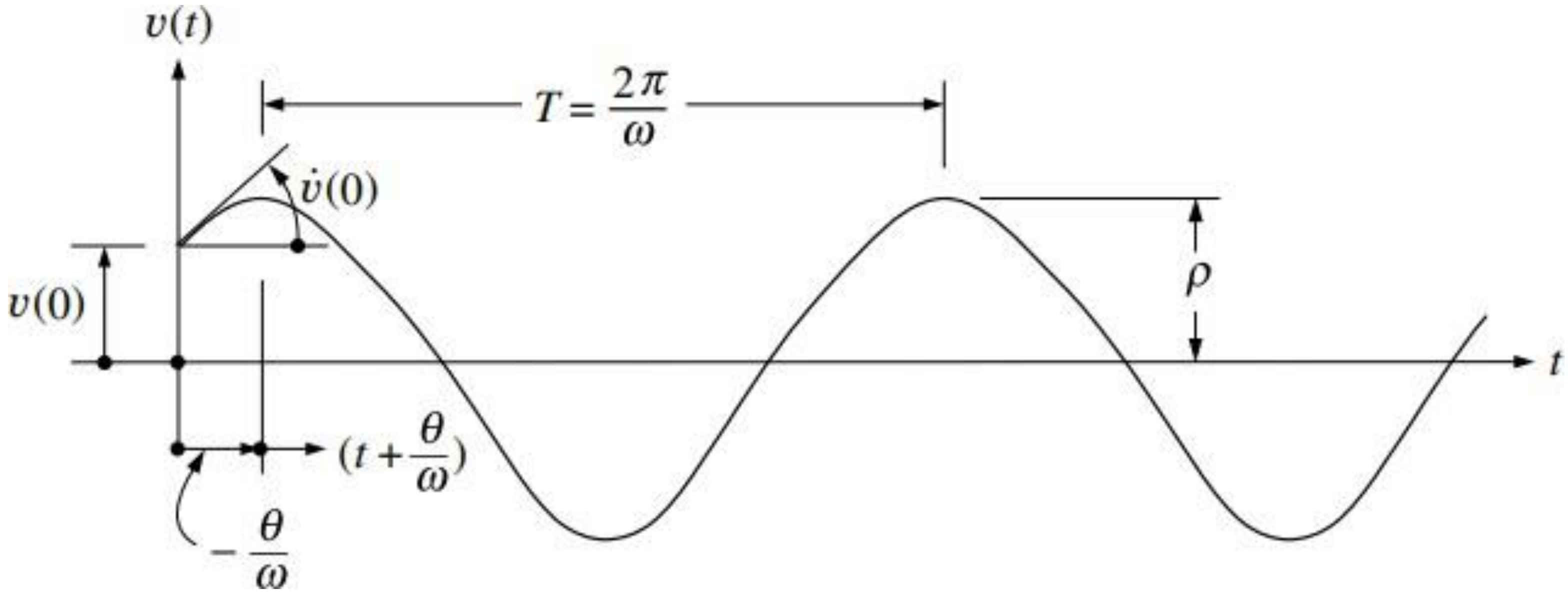
Substituting $\omega^2 \equiv k/m$

Cyclic frequency referred to as frequency of motion is given by

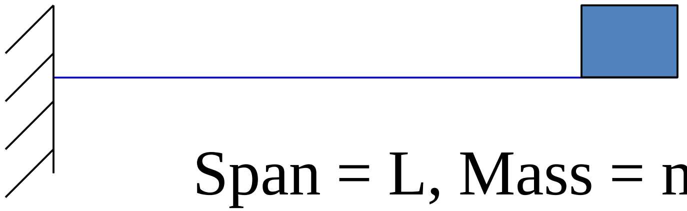
$f = \omega/2\pi$ Cycles/sec

And after inverting the same we get

$1/f = 2\pi\ \omega = T$ Time to complete one cycle of motion

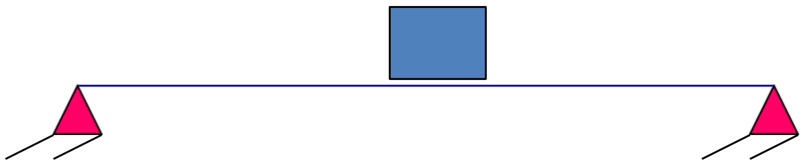


CANTILEVER BEAM WITH POINT MASS



Stiffness = $3EI/L^3$

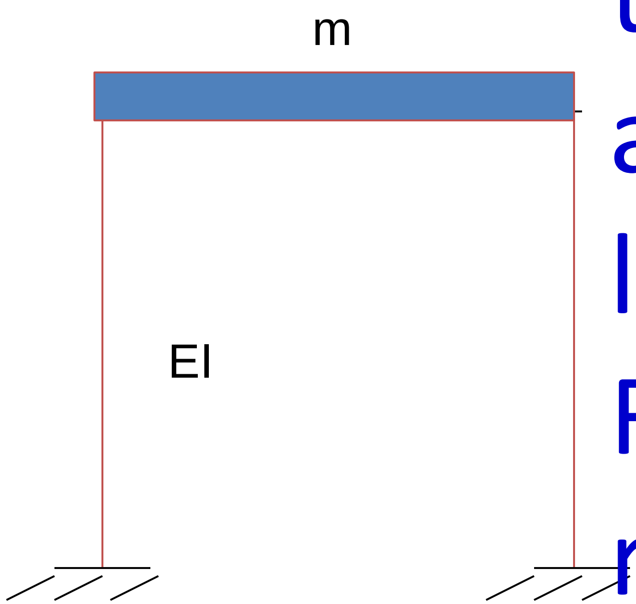
Span = L, Mass = m, EI = const



Stiffness = $48EI/L^3$

SIMPLY SUPPORTED BEAM WITH POINT MASS

Portal Fram



$K = 2 \times 12\, EI/L^3$

Forced vibration

- Governing equation

Governing equation of motion is

$$Md^2y/dt^2 + Ky = F_0 f(t)$$

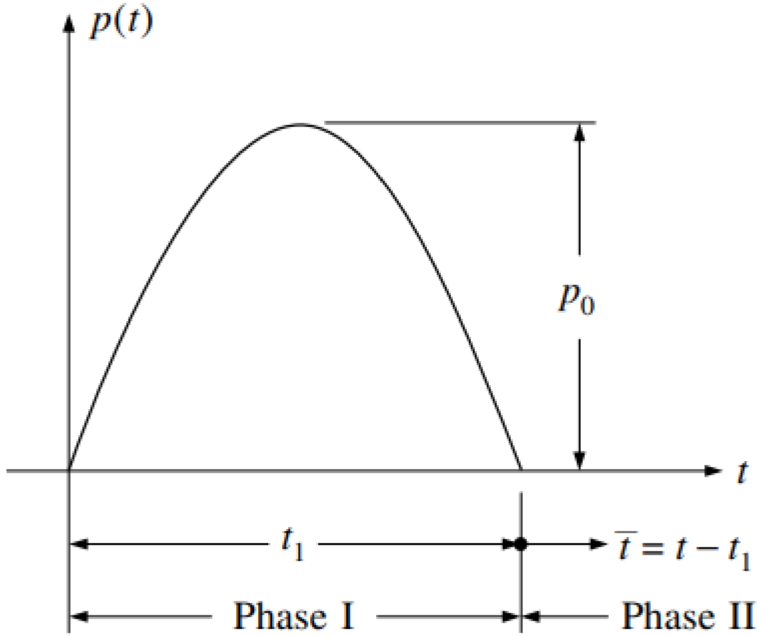
And the general solution is

$$Y = A \cos \omega t + B \sin \omega t + 1/m\omega * \int F(t) Sin\omega(t-\tau) d\tau$$

Sinusoidal pulse

$$p_o \sin \overline{\omega} t$$

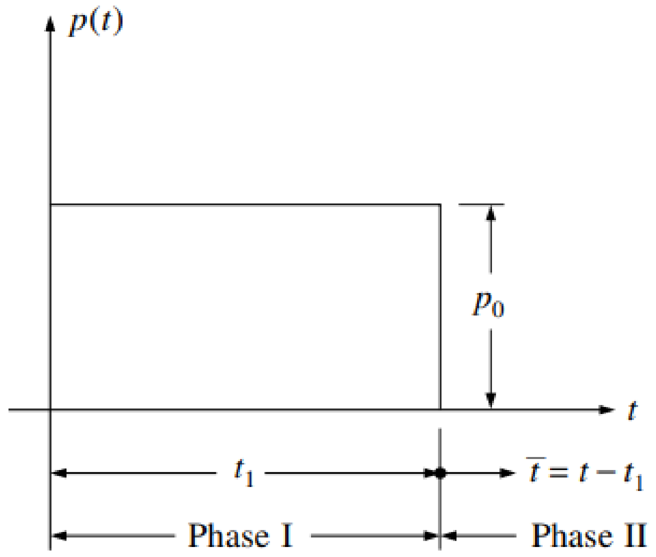
$$R(t) = \left[\frac{1}{1-\beta^2} \right] (\sin \overline{\omega} t - \beta \sin \omega t)$$



Rectangular pulse

ii. Solution of equation of motion

$Y = F_0/k\,(1 - \cos \omega t)$

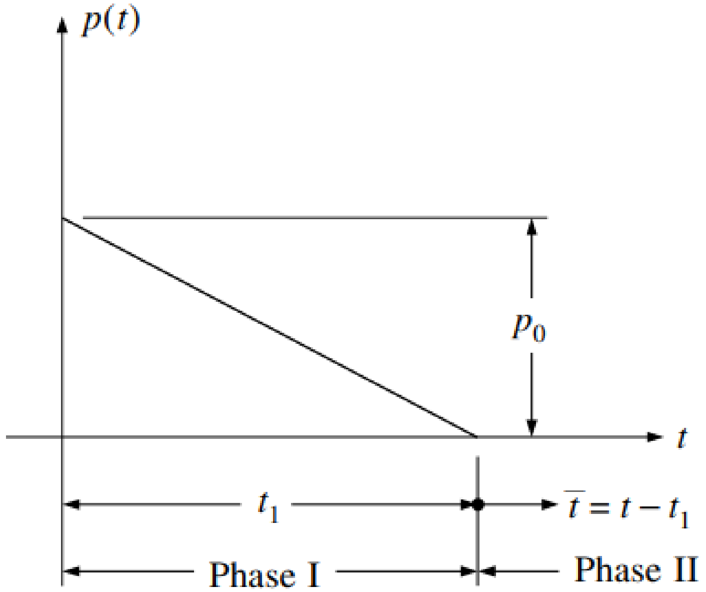


Triangular Pulse

Triangular pulse (Explosion) $F = F_0(1\text{-}t/t_d)$

Solution of equation of motion is

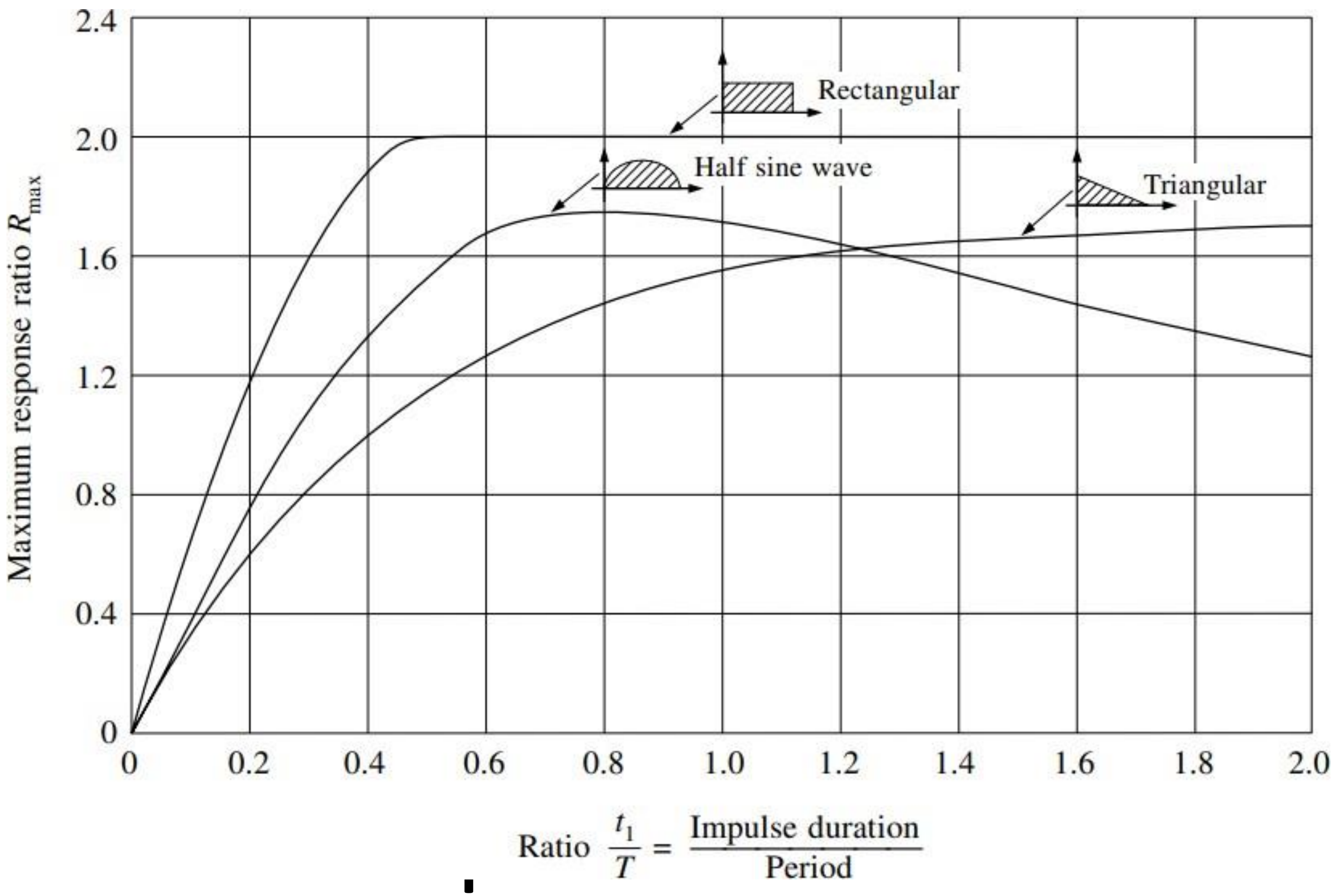
$Y = F_0/k\ (1\text{- Cos}\ \omega t) + F_0/Kt_d((\text{Sin}\ \omega t/)\omega\ \text{-}\ t)$



R

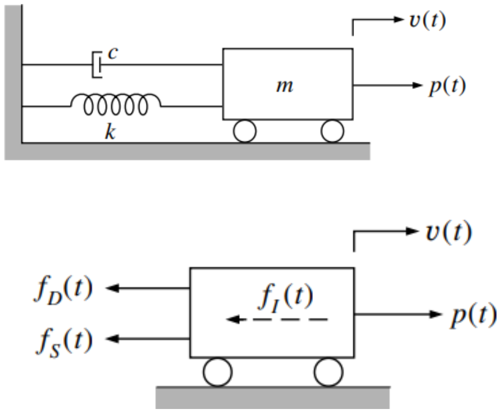
e

s



Rectangular
Half sine wave
Triangular
Maximum response ratio R_{max}
2.4
2.0
1.6
1.2
0.8
0.4
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Ratio $\frac{t_1}{T} = \frac{\text{Impulse duration}}{\text{Period}}$

Undamped system under harmonic force



$$m\,\ddot{v}(t) + c\,\dot{v}(t) + k\,v(t) = P_0 \sin pt$$

$$\beta = \frac{\bar{p}}{\omega}$$ frequency ratio

General solution of the equation under undamped condition is given as

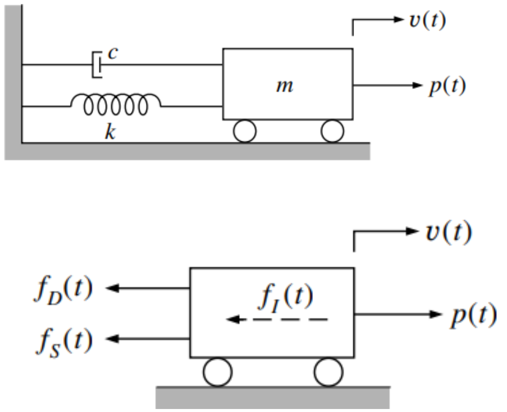
$$v(t) = v_c(t) + v_p(t) = A\,\cos\omega t + B\,\sin\omega t + \frac{p_o}{k}\left[\frac{1}{1-\beta^2}\right] \sin pt$$

Dynamic magnification factor under undamped condition is $\frac{1}{1-\beta^2}$

Damped system under harmonic force

$$\ddot{v}(t) + 2\,\xi\,\omega\,\dot{v}(t) + \omega^2\,v(t) = \frac{p_o}{m}\ \text{sin pt}$$

$$c/m = 2\,\xi\,\omega$$



General solution of the equation under damped condition is given as

$$v(t) = [A\,\cos\omega_D t + B\,\sin\omega_D t]\ \exp(-\xi\omega t) \qquad \beta = \frac{p}{\omega}\ \text{frequency ratio}$$

$$+\frac{p_o}{k}\left[\frac{1}{(1-\beta^2)^2 + (2\xi\beta)^2}\right]\left[(1-\beta^2)\,\text{Sin pt} - 2\xi\beta\,\cos \text{Pt}\right]$$

A first term in above solution damps out in course of time as t $\longrightarrow \infty$

Damped system under harmonic force

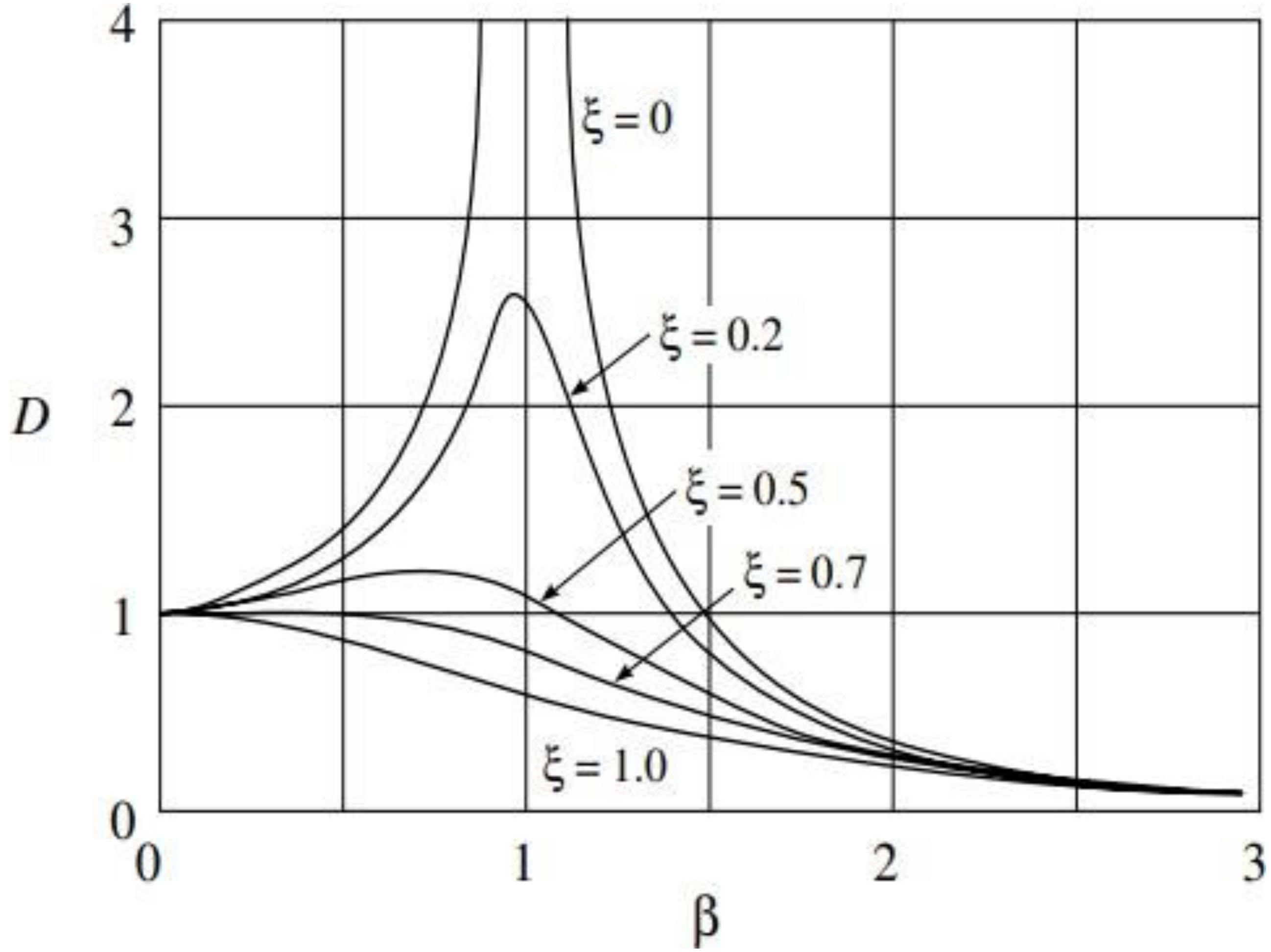
Steady state response under harmonic loading is given as

$$\frac{p_o}{k}\left[\frac{1}{(1-\beta^2)^2+(2\xi\beta)^2}\right]\left[(1-\beta^2)\text{Sin pt} \;-2\xi\beta\cos \text{Pt}\right]$$

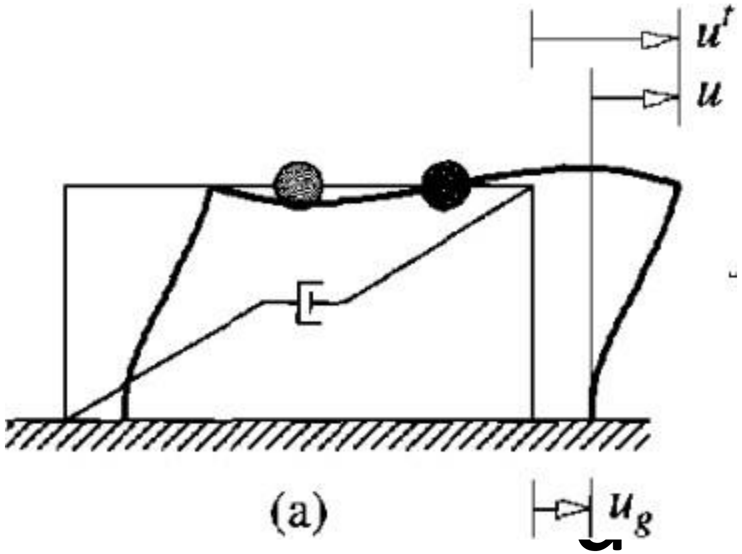
The dynamic magnification factor under damped state is given as

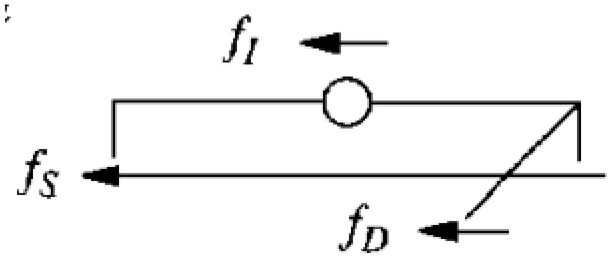
$$\text{D} = \frac{Maximum\ Dynamic\ deflection}{Maximum\ static\ deflection} = \left[(1-\beta^2)^2+(2\xi\beta)^2\right]^{-1/2}$$

Variation of Dynamic magnification factor with damping and frequency ratio



S
t
r





r
e
s

$$u^t(t) = u(t) + u_g(t)$$

Total disp = relative disp + ground disp

Equation of motion is given by

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

Strucures

Dividing above equation by m we get the following equation

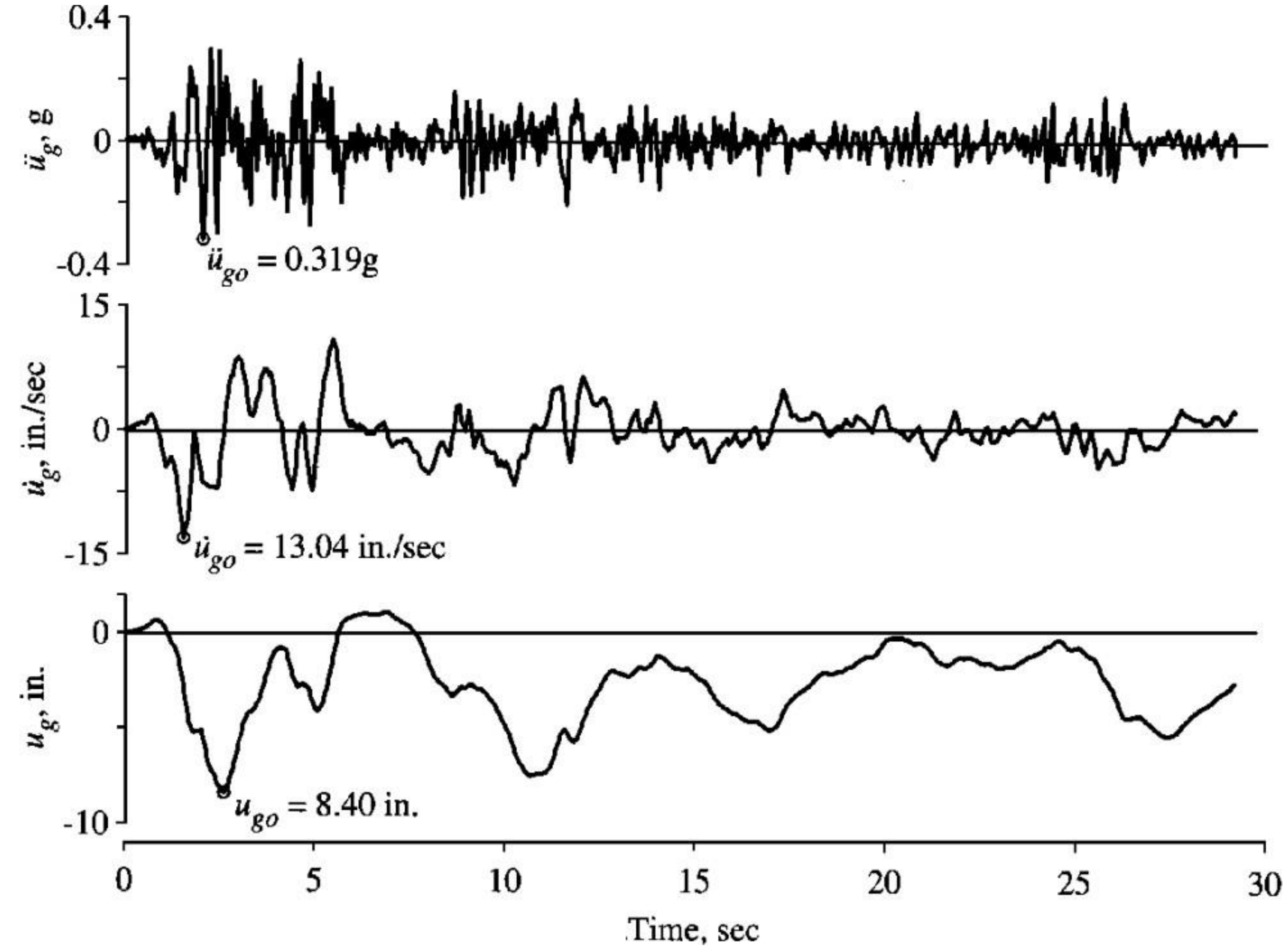
$$\ddot{u} + 2\zeta\omega_n \dot{u} + \omega_n^2 u = -\ddot{u}_g(t)$$

It is clear that for a given $\ddot{u}_g(t)$, the deformation response $u(t)$ of the system depends only on the natural frequency ω_n or natural period T_n of the system and its damping ratio ζ;

In other words $u \equiv u(t, T_n, \zeta)$.

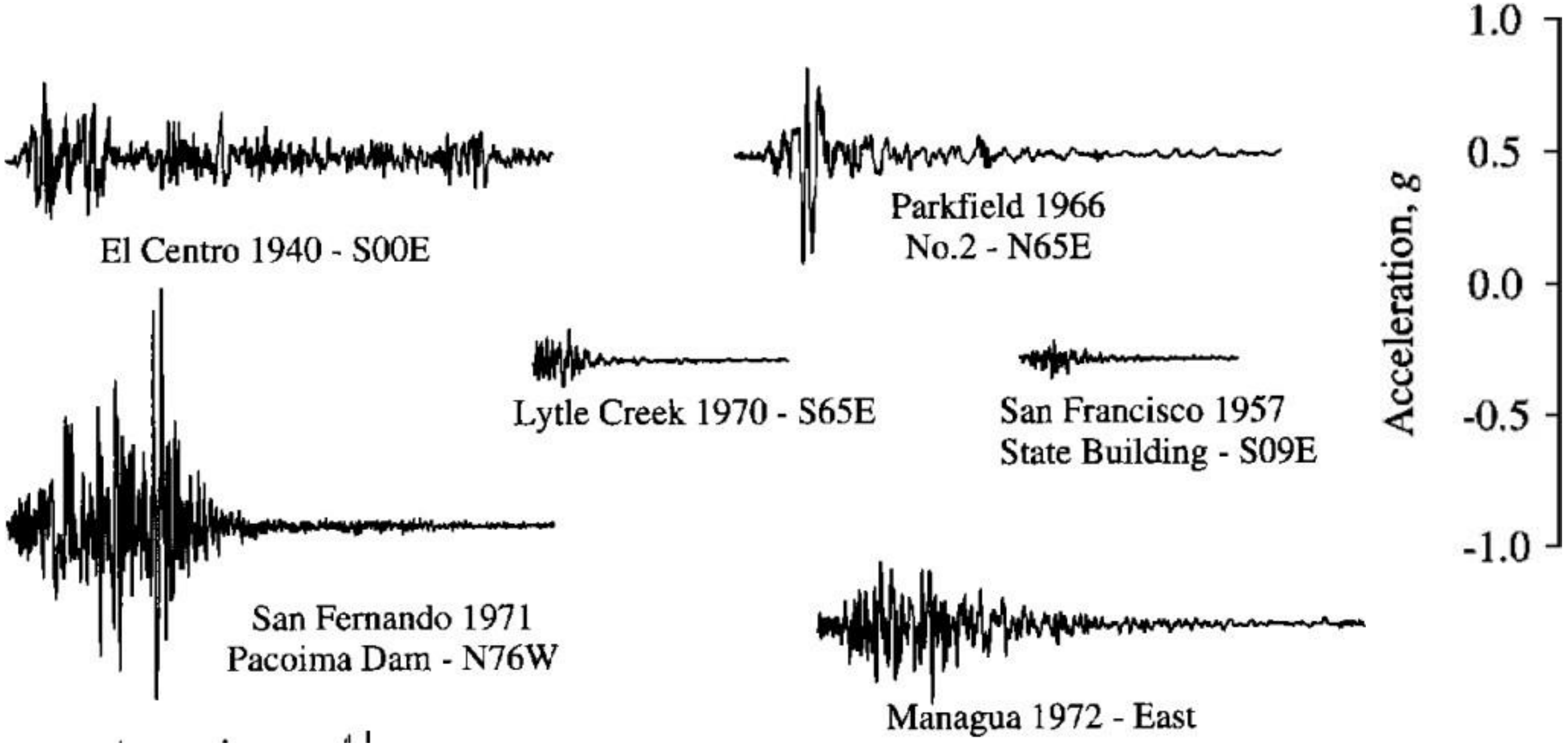
Thus any two systems having the same values of T_n and ζ will have the same deformation response $u(t)$ even though one system may be more massive than the other or one may be stiffer than the other.

Typical Ground Motion Records



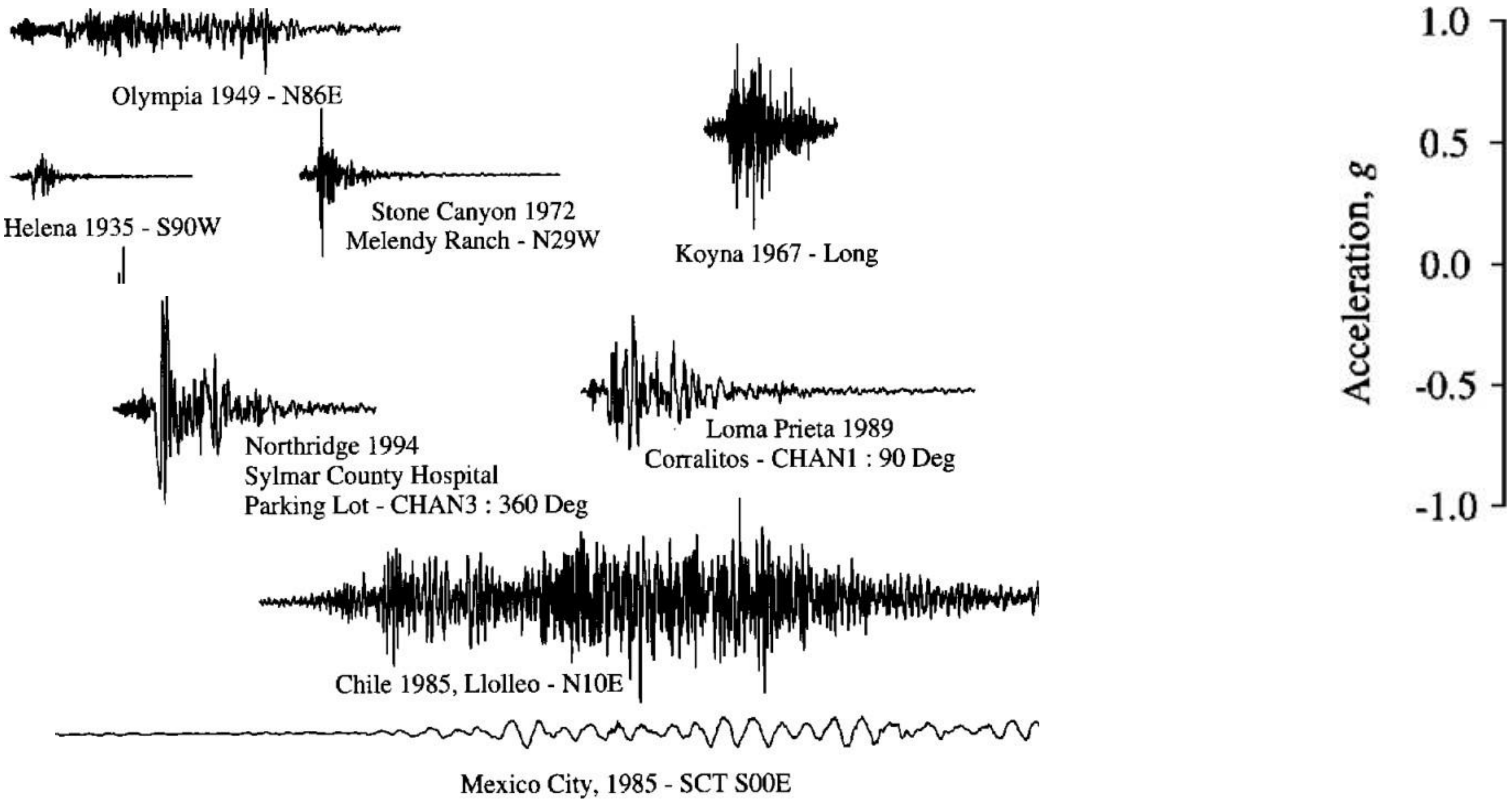
North–south component of horizontal ground acceleration recorded at the Imperial Valley Irrigation District substation, El Centro, California, during the Imperial Valley earthquake of May 18, 1940. The ground velocity and ground displacement were computed by integrating the ground acceleration.

Earthquake response of linear system



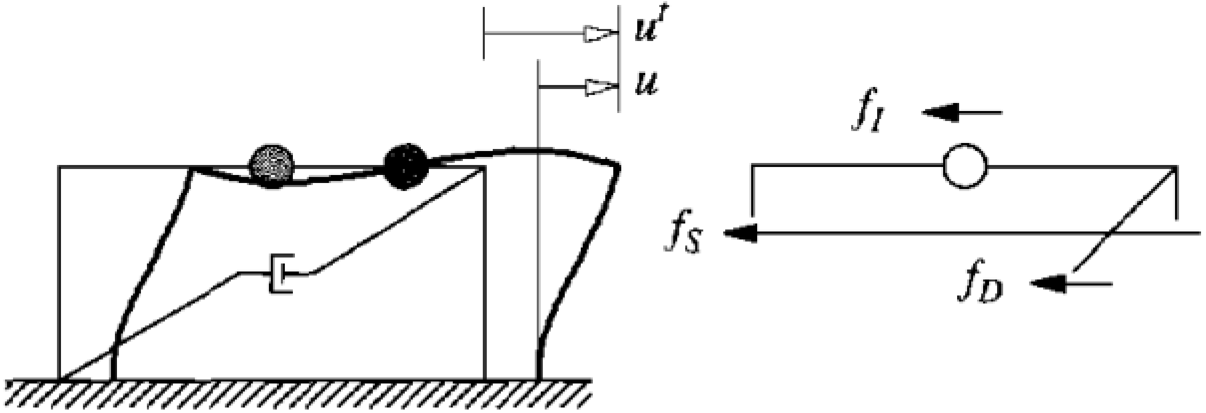
Ground motion recorded during several earthquakes

Earthquake response of linear system contd.



Ground motion recorded during several earthquakes

Simple system under ground motion

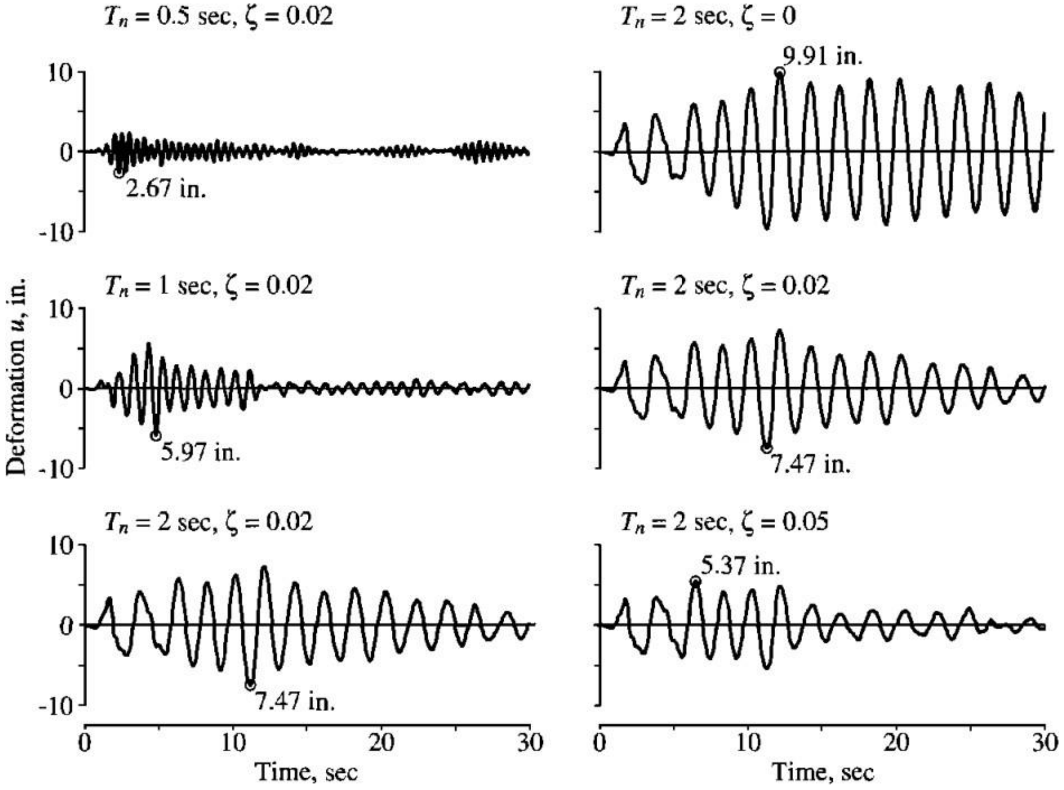


Equation of motion under the three forces is

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

Response of SDOF to ground motion

For a given ground motion $\ddot{u}_g(t)$, the deformation response $u(t)$ of an SDF system depends only on the natural vibration period of the system and its damping ratio.



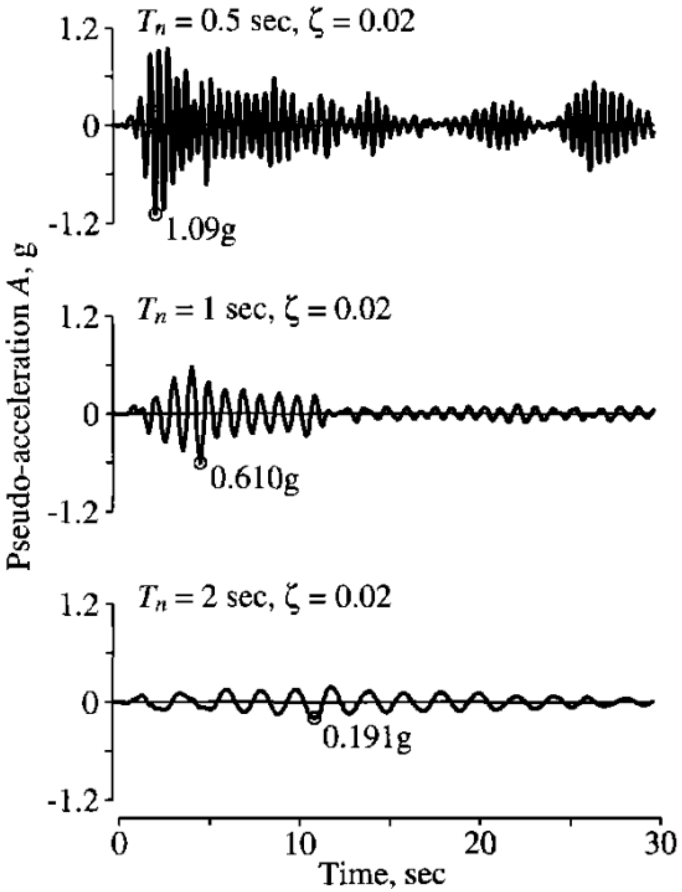
Deformation response of SDF systems to El Centro ground motion.

Pseudo acceleration response

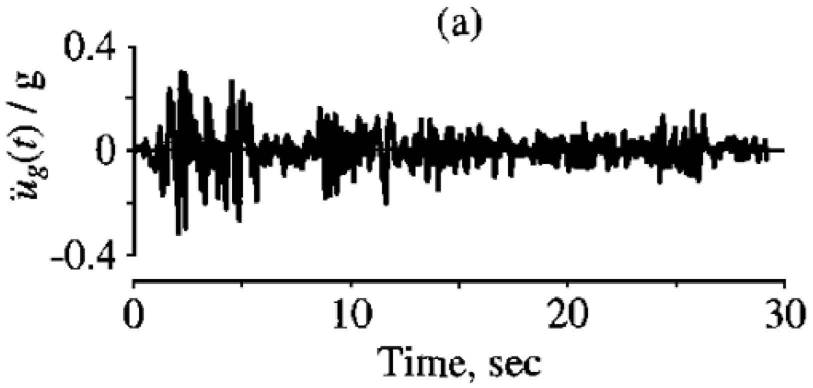
Pseudo acceleration = displacement x $\omega_n^2 = (2\pi/T_n)^2$

$$V = \omega_n D = \frac{2\pi}{T_n} D$$

$$A = \omega_n^2 D = \left(\frac{2\pi}{T_n}\right)^2 D$$

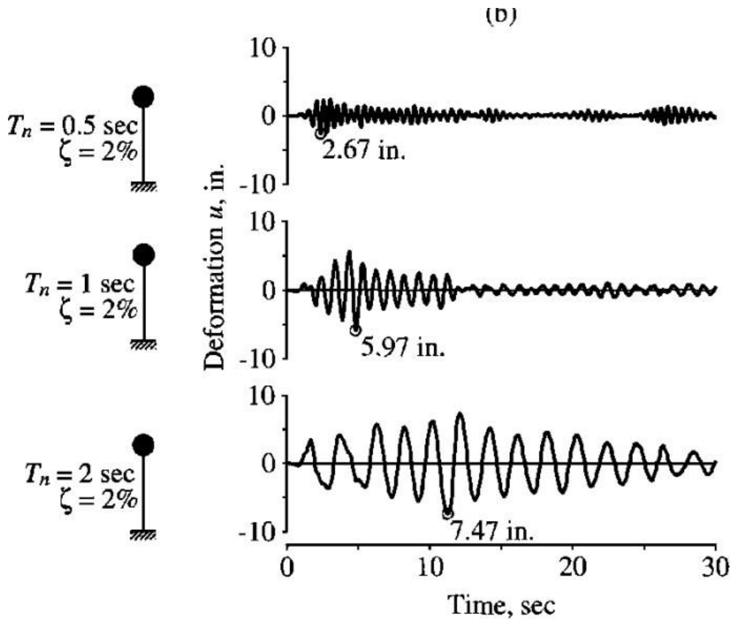


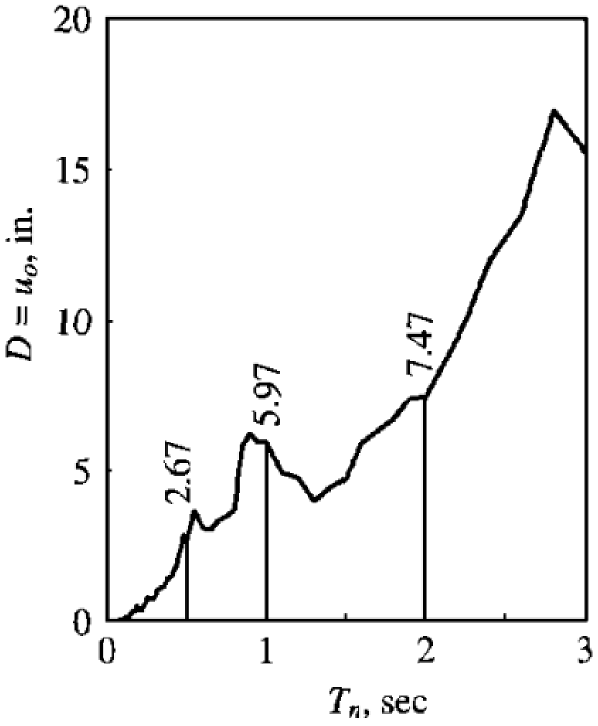
Response spectrum



El-centro ground motion

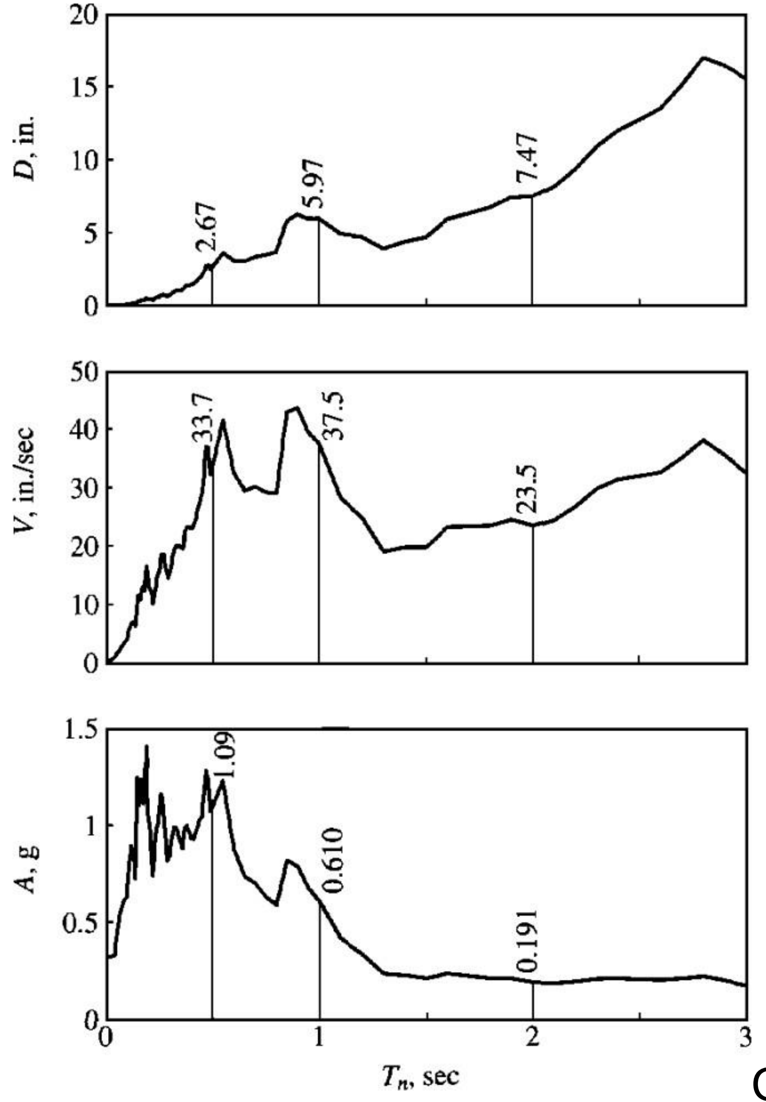
Deformation response spectrum





Deformation response

Pseudo velocity and pseudo acceleration spectra



Deformation Response Spectrum
Gives force F = k Δ

Pseudo-velocity Response Spectrum

$$V = \omega_n D = \frac{2\pi}{T_n} D$$

Gives energy

$$E_{So} = \frac{ku_o^2}{2} = \frac{kD^2}{2} = \frac{k(V/\omega_n)^2}{2} = \frac{mV^2}{2}$$

Pseudo- acceleration Response Spectrum

$$A = \omega_n^2 D = \left(\frac{2\pi}{T_n}\right)^2 D$$

Gives base shear $V_{bo} = f_{So} = mA$ $= m\omega_n^2 D$

Combined D–V–A Spectrum

Each of the deformation, pseudo-velocity and pseudo acceleration response spectra for a given ground motion contain the same information

The three spectra are simply different ways of presenting the same information on structural response

Displacement response spectra gives max displacement
Velocity response spectra gives max energy stored
Acceleration response spectra gives max equivalent static force/base shear

Three spectra are inter related as follows

$$\frac{A}{\omega_n} = V = \omega_n D \quad \text{or} \quad \frac{T_n}{2\pi} A = V = \frac{2\pi}{T_n} D$$

4way log scale paper

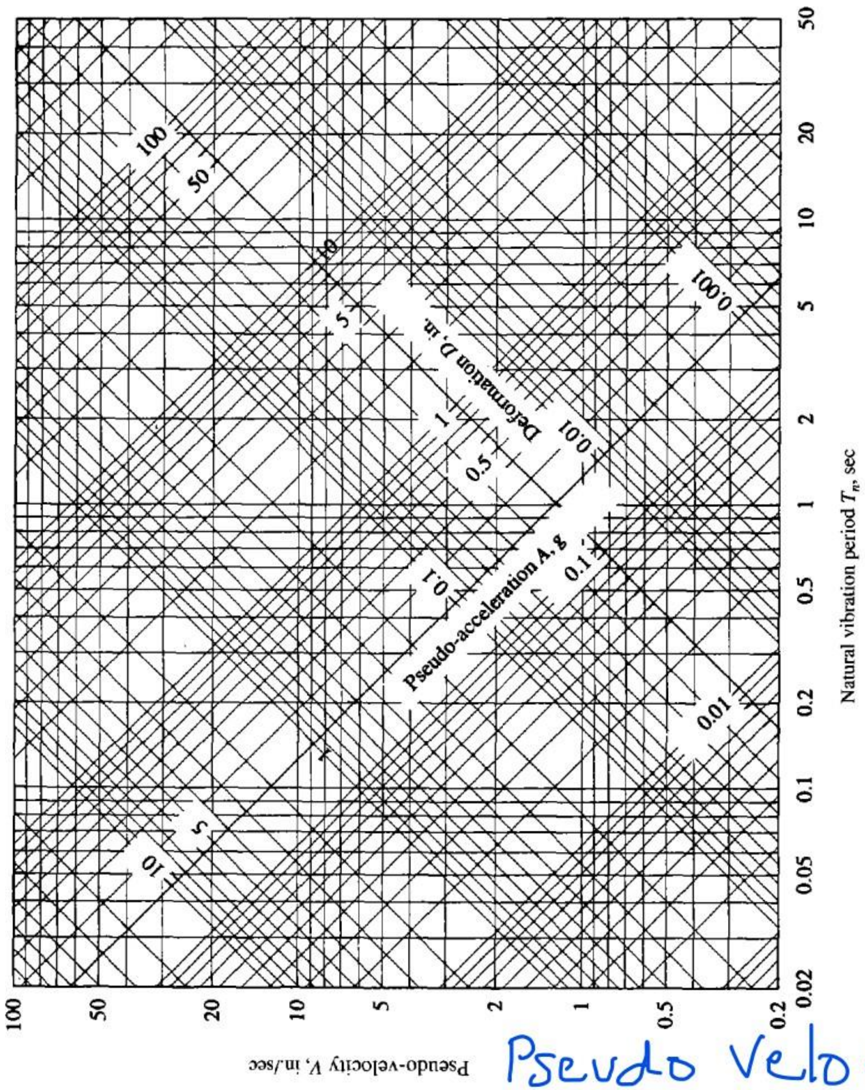


Figure A6.1 Graph paper with four-way logarithmic scales.

Log scale
T_n

Pseudo Velocity
V in/sec

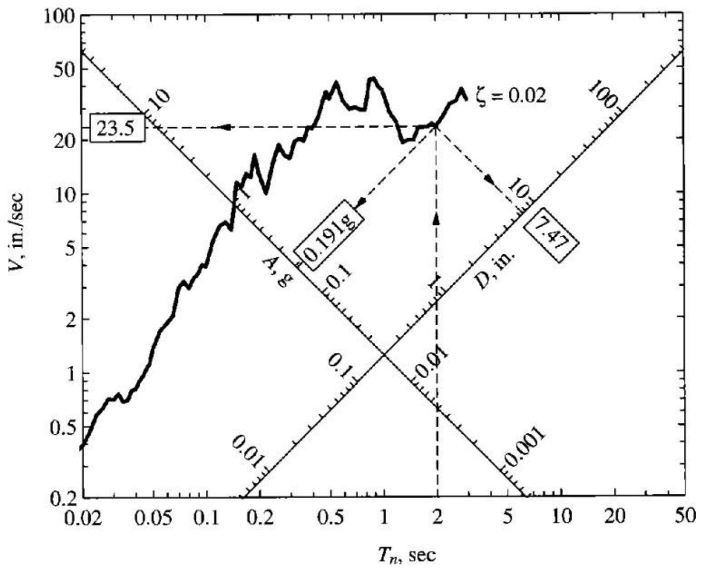
Log scale
V

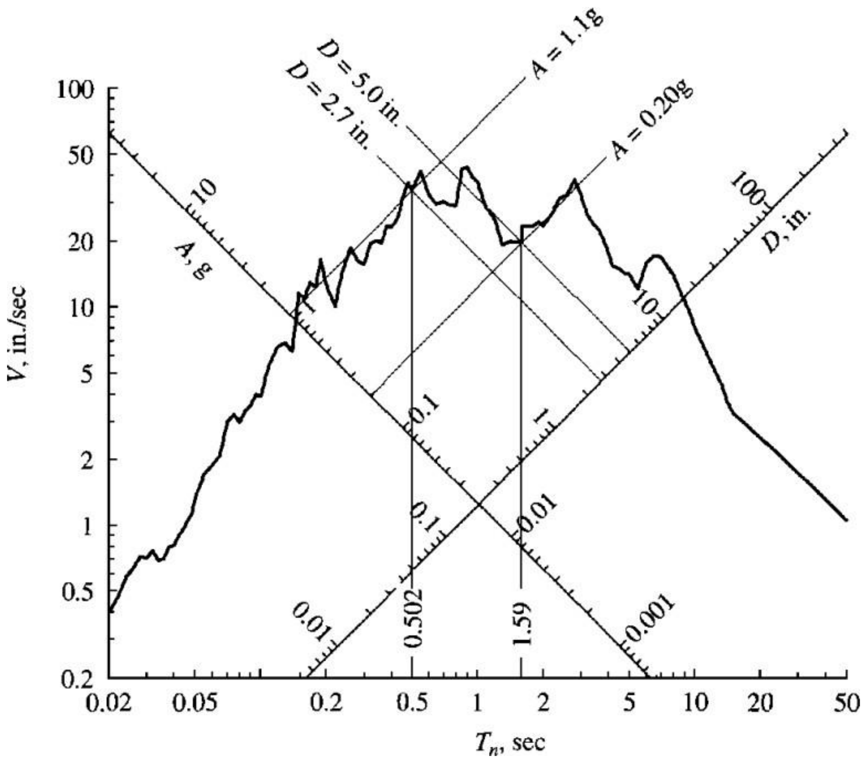
3 way response spectra

Three spectra quantities are related to each other as follows

$$\frac{A}{\omega_n} = V = \omega_n D \quad \text{or} \quad \frac{T_n}{2\pi} A = V = \frac{2\pi}{T_n} D$$

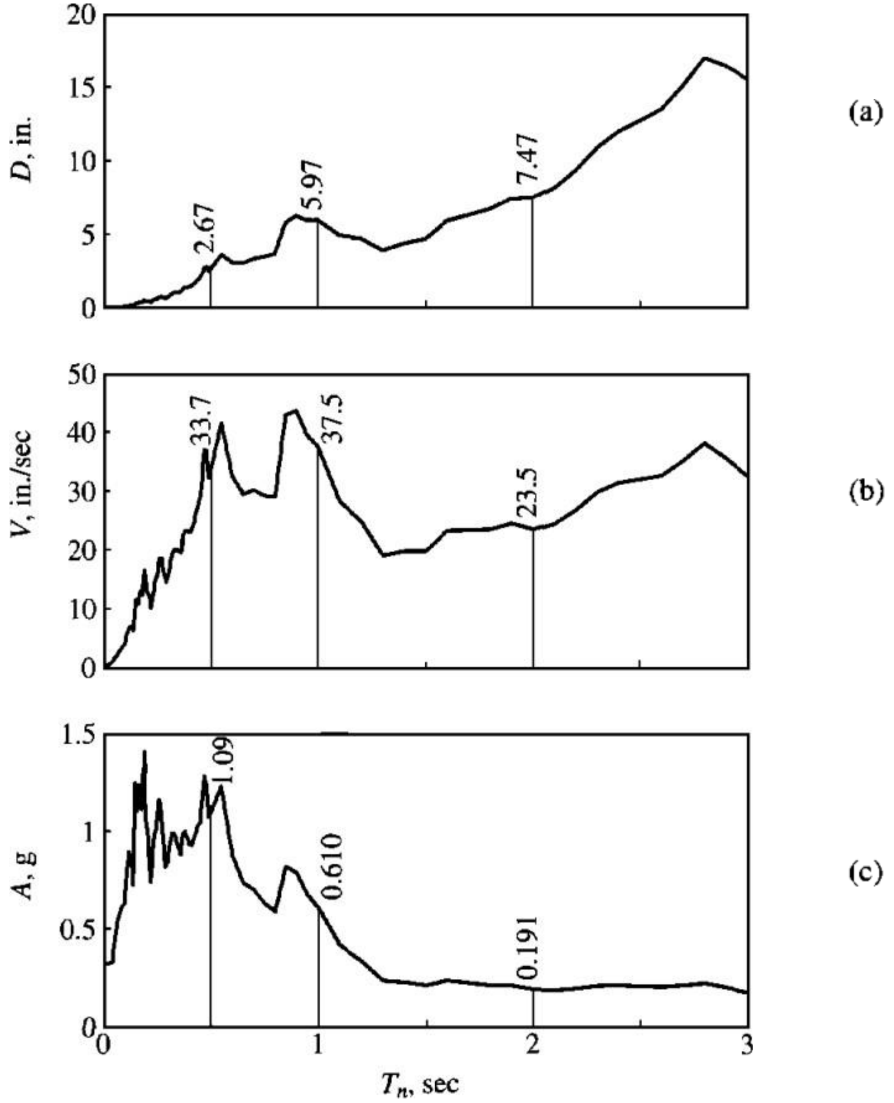
Combined *D–V–A* Spectrum



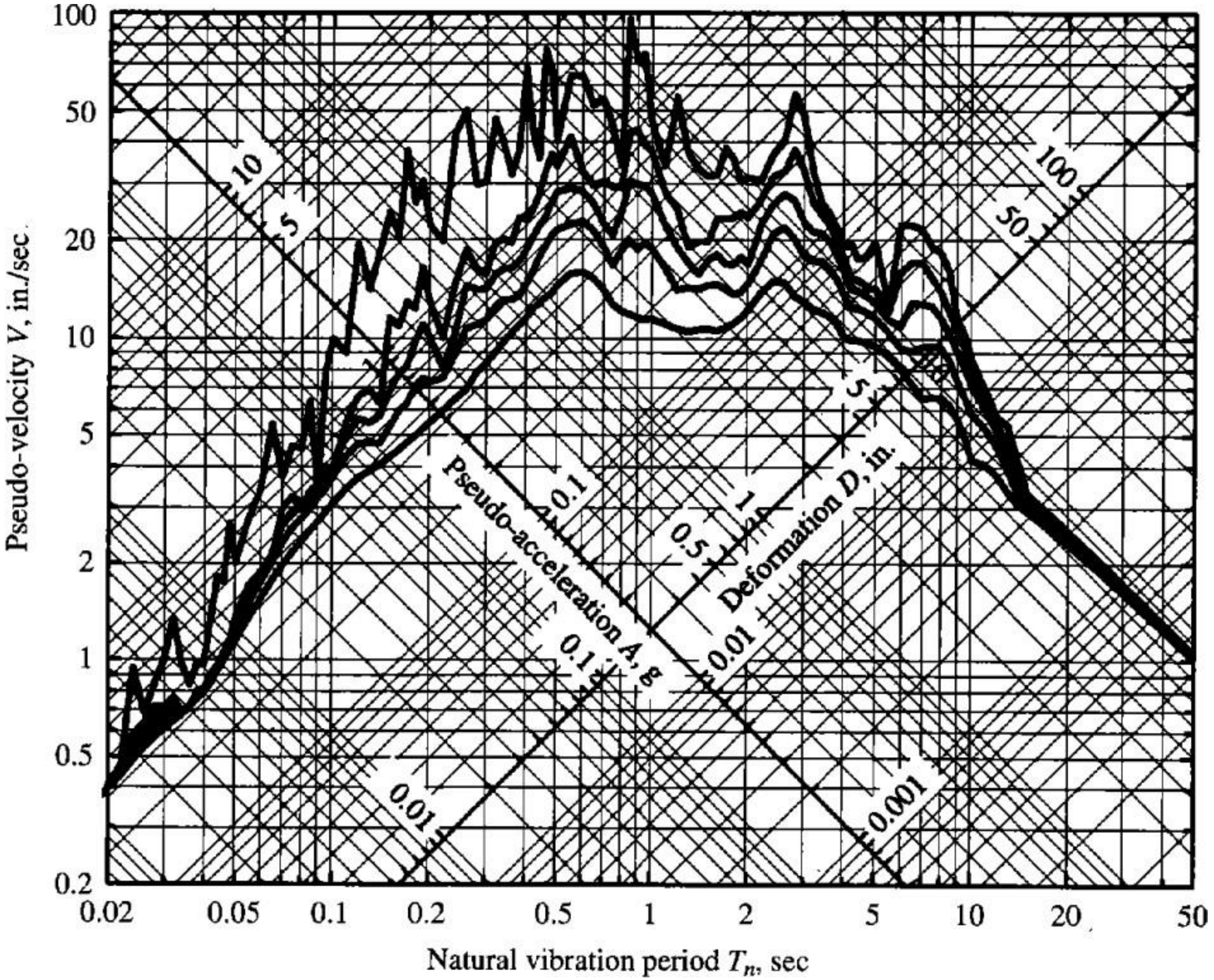


Construction of Response spectrum

1. Numerically define the ground acceleration $\ddot{u}_g(t)$; typically, the ground motion ordinates are defined every 0.02 sec.
2. Select the natural vibration period T_n and damping ratio ζ of a SDF system.
3. Compute the deformation response $u(t)$ of this SDF system due to the ground motion $\ddot{u}_g(t)$ by any of the numerical methods
4. Determine u_o, the peak value of $u(t)$.
5. The spectral ordinates are $D = u_o$, $V = (2\pi/T_n)D$, and $A = (2\pi/T_n)^2 D$.
6. Repeat steps 2 to 5 for a range of T_n and ζ values covering all possible systems of engineering interest.
7. Present the results of steps 2 to 6 graphically to produce three separate spectra or a combined spectrum



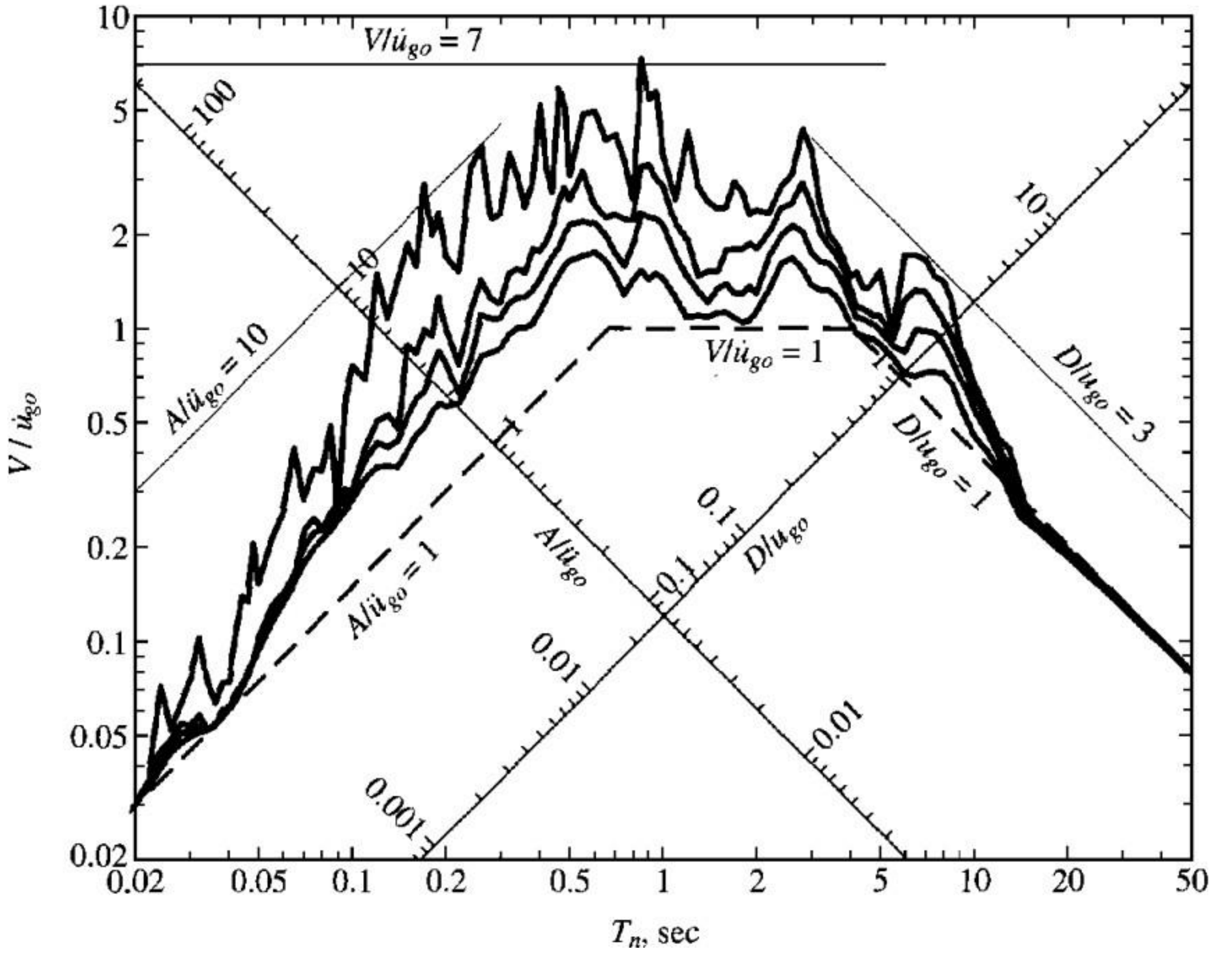
Response spectra ($\zeta \simeq 0.02$) for El Centro ground motion: (a) deformation response spectrum; (b) pseudo-velocity response spectum; (c) pseudo-acceleration response spectrum.



Combined D–V–A response spectrum for El Centro ground motion;

$\zeta = 0, 2, 5, 10,$ and 20%.

Normalized Response spectrum

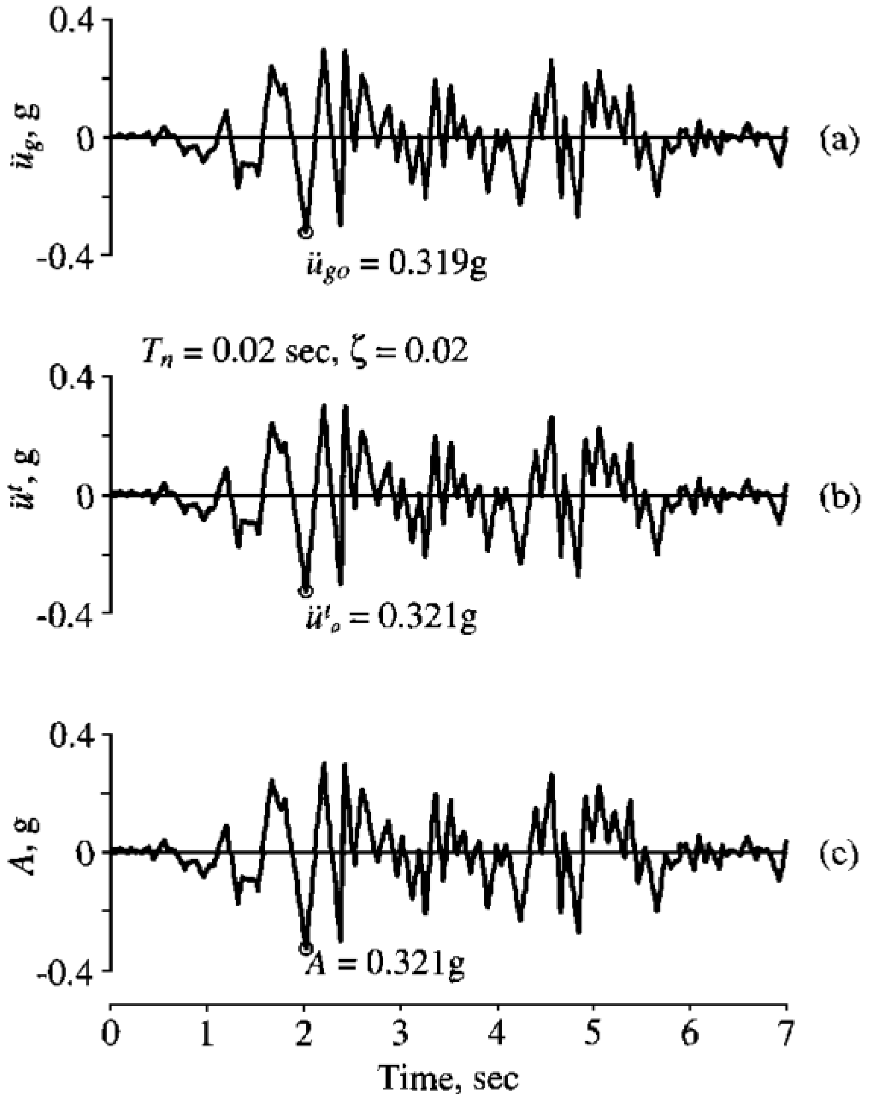


Response spectrum for El Centro ground motion plotted with normalized scales $A/\ddot{u}_{go}$, $V/\dot{u}_{go}$, and D/u_{go}; $\zeta = 0, 2, 5,$ and 10%.

Response of a very rigid system   T = 0.02

For this system, the structure is very rigid and hence the mass acceleration
Will be same as ground acceleration

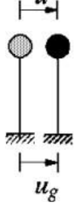
Response of a very rigid system   T = 0.02



(a) El Centro ground acceleration;

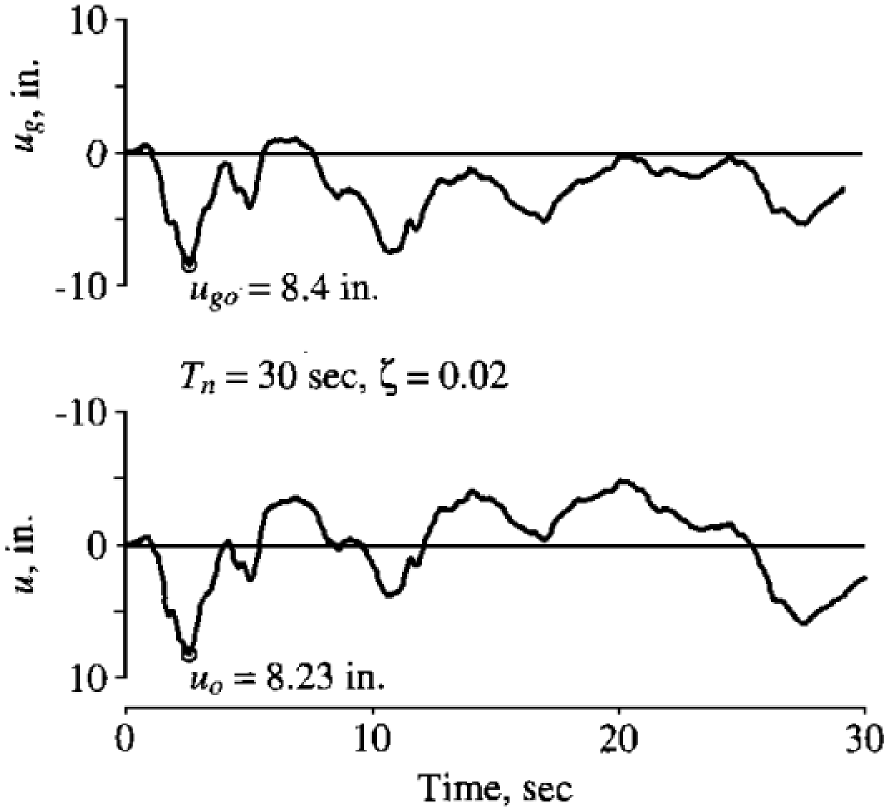
(b) total acceleration response of an SDF system with $T_n = 0.02$ sec and $\zeta = 2\%$;

(c) pseudo-acceleration response of the same system;



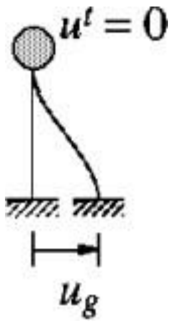
rigid system.

Response of a very Flexible system   T = 30 sec



El Centro ground displacement;

deformation response of SDF system with $T_n = 30$ sec and $\zeta = 2\%$

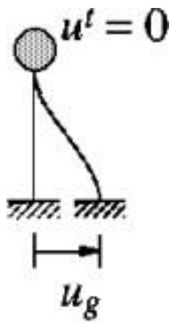


very flexible system.

Response of a very flexible system   T >15 sec

For this system, structure is very flexible, and hence the mass would
Remain stationary resulting into

$$u(t) \simeq -u_g(t)$$



Response of short period system
0.035 <T 0.5 sec

For short-period systems with T_n between $T_a = 0.035$ sec and $T_c = 0.50$ sec

Acceleration of mass exceeds ground acceleration and Magnification depends on Tn and damping

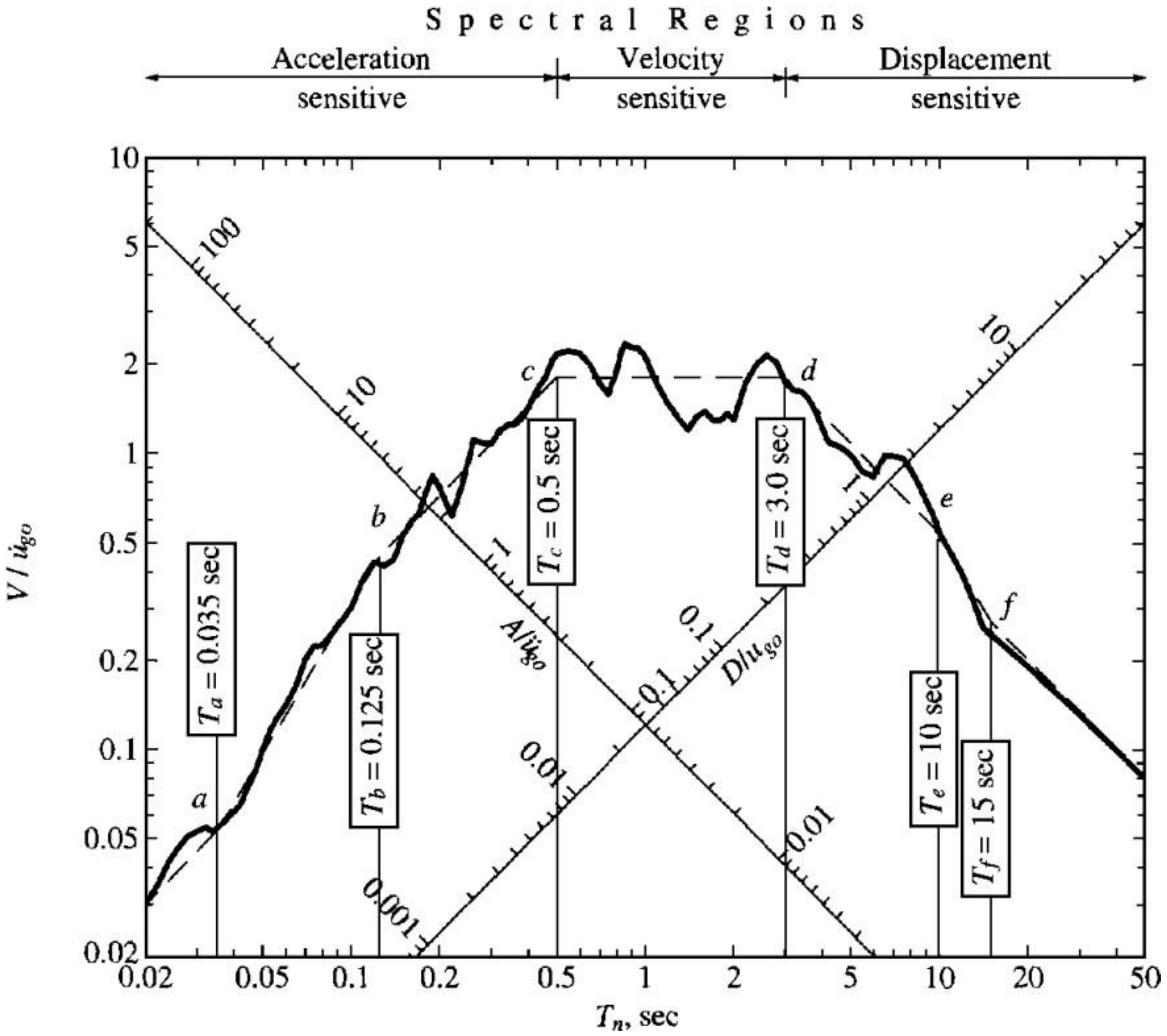
For 0.125 < Tn < 0.5 the mass acceleration is constant equal to ground acceleration magnified by a factor depending ondamping

For 0.5 < Tn < 0.3 the mass velocity is constant equal to ground velocity magnified by a factor depending damping

For 3 < Tn < 15 the mass Displacement is greater than ground Displacement and magnification depends on Tn and damping

For 3 < Tn < 10 the mass Displacement is constant equal to ground Displacement magnified by a factor depending on damping

Spectrum is divided into three zones



Solid line – response spectrum for El centro earthquake
Dashed line – idealized response spectrum for El centro earthquake

Elastic Design spectrum

Generally response spectrum of each recorded earthquake ground motion Is different

Elastic design spectrum is used to design new structures for future earthquake

The Design spectrum should be representative of ground motion recorded at site during past earthquake

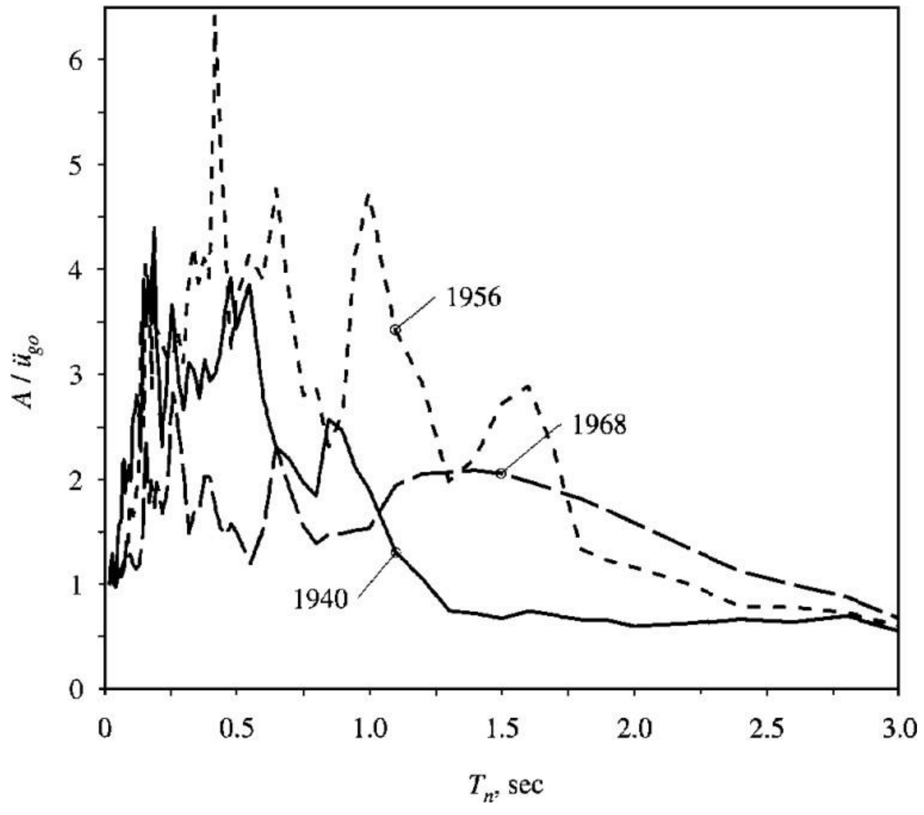
If such record is not available, the design spectrum should be based onThe record

If such record is not available, the design spectrum should be based on the record available at other site under similar condition

The factors to be matched are

1. Magnitude of earthquake
2. Distance of site from source of earthquake
3. Fault mechanism
4. Geology of travel path
5. Local soil conditions

If such records in sufficient numbers are not available then statistical approach is necessary to consider available records and do some averaging of results



Response spectra of Imperial valley earthquakes, El centro California
18 may 1940
9 February 1956
8 April 1968

Y axis is normalized mass acceleration
= mass acceleration/ground acceleration

Typical Seismic ground motion